

Spring
Scheme of learning

Year 5

White Rose
MATHS

#MathsEveryoneCan

Spring Block 1

Multiplication and division B

Small steps

Step 1

Multiply up to a 4-digit number by a 1-digit number

Step 2

Multiply a 2-digit number by a 2-digit number (area model)

Step 3

Multiply a 2-digit number by a 2-digit number

Step 4

Multiply a 3-digit number by a 2-digit number

Step 5

Multiply a 4-digit number by a 2-digit number

Step 6

Solve problems with multiplication

Step 7

Short division

Step 8

Divide a 4-digit number by a 1-digit number



Small steps

Step 9

Divide with remainders

Step 10

Efficient division

Step 11

Solve problems with multiplication and division



Multiply up to a 4-digit number by a 1-digit number

Notes and guidance

In Year 4, children used the formal written method to multiply numbers with up to three digits by a 1-digit number. This small step builds on this learning and extends the formal written method for short multiplication to multiplying 4-digit numbers by a 1-digit number.

Place value counters in place value charts are used to model the structure of the formal method, enabling children to gain a greater understanding of the abstract procedure. Children continue to use counters to exchange groups of 10 ones for 1 ten and this is extended to include exchanging 10 tens for 1 hundred, 10 hundreds for 1 thousand and 10 thousands for 1 ten-thousand.

Children can use their knowledge of rounding and multiplying by multiples of 10 to find estimates to the answers, as a check that their calculated answers are sensible.

Things to look out for

- Children may make errors when multiplying by zero.
- Children may omit the exchange or include the exchange in an incorrect column in the formal written method.
- Children may write more than one digit in a single column rather than make an exchange.

Key questions

- How does multiplication link to addition?
- How can you use counters to represent 284×3 ?
- How does the written method match the representation?
- Which column do you start with?
- Do you need to make an exchange?
- How could you estimate the answer to check your calculation?
- What is the same and what is different about multiplying a 4-digit number by a 1-digit number and multiplying a 3-digit number by a 1-digit number?

Possible sentence stems

- _____ ones \times _____ = _____ ones + _____ tens
- _____ tens \times _____ = _____ tens + _____ hundreds
- _____ hundreds \times _____ = _____ hundreds + _____ thousands
- _____ thousands \times _____ = _____ thousands + _____ ten-thousands

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Multiply up to a 4-digit number by a 1-digit number

Key learning

- Complete the sentences to describe the multiplication.

| Thousands | Hundreds | Tens | Ones |
|-------------|----------|------|-------|
| 1,000 1,000 | 100 | | 1 1 1 |
| 1,000 1,000 | 100 | | 1 1 1 |
| 1,000 1,000 | 100 | | 1 1 1 |

There are _____ ones altogether.
 There are _____ tens altogether.
 There are _____ hundreds altogether.
 There are _____ thousands altogether.
 $2,103 \times 3 =$ _____

- There are 1,152 seats in a cinema.
 The cinema is fully booked for three showings of a film.
 How many people go to the film altogether?

| Th | H | T | O |
|-------|-----|----------|-----|
| 1,000 | 100 | 10 10 10 | 1 1 |
| 1,000 | 100 | 10 10 10 | 1 1 |
| 1,000 | 100 | 10 10 10 | 1 1 |

| | | | | | | | | |
|--|---|-------|---|---|---|--|--|--|
| | | | | | | | | |
| | | 1 | 1 | 5 | 2 | | | |
| | x | | | | 3 | | | |
| | | _____ | | | | | | |
| | | _____ | | | | | | |

- Ms Fisher earns £1,325 per week.
 How much does she earn in 4 weeks?

| Th | H | T | O |
|-------|-------------|-------|-------|
| 1,000 | 100 100 100 | 10 10 | 1 1 1 |
| 1,000 | 100 100 100 | 10 10 | 1 1 1 |
| 1,000 | 100 100 100 | 10 10 | 1 1 1 |
| 1,000 | 100 100 100 | 10 10 | 1 1 1 |

| | | | | | | | | |
|--|---|-------|---|---|---|--|--|--|
| | | | | | | | | |
| | | 1 | 3 | 2 | 5 | | | |
| | x | | | | 4 | | | |
| | | _____ | | | | | | |
| | | _____ | | | | | | |

- Complete the calculations.

| | | | | | | | | |
|--|---|-------|---|---|---|--|--|--|
| | | | | | | | | |
| | | 2 | 3 | 2 | 3 | | | |
| | x | | | | 4 | | | |
| | | _____ | | | | | | |
| | | _____ | | | | | | |

| | | | | | | | | |
|--|---|-------|---|---|---|--|--|--|
| | | | | | | | | |
| | | 2 | 4 | 5 | 1 | | | |
| | x | | | | 2 | | | |
| | | _____ | | | | | | |
| | | _____ | | | | | | |

| | | | | | | | | |
|--|---|-------|---|---|---|--|--|--|
| | | | | | | | | |
| | | 1 | 3 | 4 | 2 | | | |
| | x | | | | 6 | | | |
| | | _____ | | | | | | |
| | | _____ | | | | | | |

Multiply up to a 4-digit number by a 1-digit number

Reasoning and problem solving

Dani works out $1,432 \times 4$

| | | | | | | |
|--|---|---|----|----|---|--|
| | | | | | | |
| | | 1 | 4 | 3 | 2 | |
| | × | | | | 4 | |
| | | 4 | 16 | 12 | 8 | |

$$1,432 \times 4 = 416,128$$

Use estimation to show that Dani must be wrong.

What mistake has Dani made?

Dani has not exchanged when she has 10 or more in the tens and hundreds columns.

$$342 \times 3 = 1,026$$

Without calculating, which is greater, 342×4 or 343×3 ?

Explain your answer.

$$342 \times 4$$

Use the clues to work out the missing numbers.

| | | | | | | | |
|--|---|--|--|--|--|---|--|
| | | | | | | | |
| | | | | | | | |
| | × | | | | | 5 | |
| | | | | | | | |

- The four digits being multiplied are consecutive numbers.
- The first two digits of the product are the same.
- The fourth and fifth digits of the product add to make the third.

Find all the possible solutions.

$$2,345 \times 5 = 11,725$$

$$4,567 \times 5 = 22,835$$

$$6,789 \times 5 = 33,945$$

Multiply a 2-digit number by a 2-digit number (area model)

Notes and guidance

In this small step, children build on their learning of multiplying by a 1-digit number and begin to multiply by a 2-digit number.

Children use the area model to multiply a 2-digit number by another 2-digit number before moving on to the formal written method in the next step. Linking the use of the area model to children's prior knowledge of arrays helps children to understand the model. They see that to find the total product, they can break the calculation down, find other products and then add them together.

Initially, the area model is represented using base 10, which will enable children to understand size, scale and place value. Once the children have a good understanding of place value within the area model, they use place value counters to work more efficiently. They then progress to using only numbers in the model.

Things to look out for

- Children may complete the area model and then forget to add together the parts.
- When moving away from using concrete resources, children may make errors when multiplying by powers of 10, for example thinking that $30 \times 40 = 120$ instead of 1,200

Key questions

- How can you partition the numbers?
- What other multiplications can you see?
- Which numbers did you multiply first?
- Once you have completed the area model, what do you need to do to find the total product of the two numbers?
- What is the same and what is different about 2×3 and 20×30 ?
- Does it matter what order you complete the area model in?

Possible sentence stems

- _____ ones \times _____ = _____ ones, so _____ tens \times _____ = _____ tens
- The products in my area model are _____, _____, _____ and _____, so the total product is _____ + _____ + _____ + _____ = _____

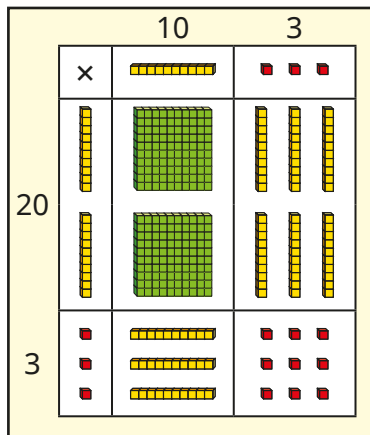
National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Multiply a 2-digit number by a 2-digit number (area model)

Key learning

- The base 10 in this area model represents 23×13



Complete the sentences.

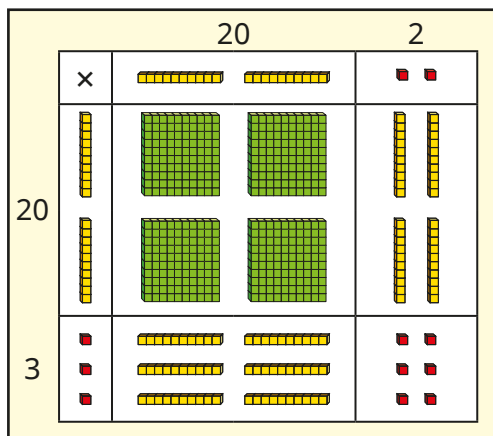
There are _____ hundreds.

There are _____ tens.

There are _____ ones.

$23 \times 13 =$ _____

- Esther uses base 10 and an area model to work out 23×22



$$23 \times 22 = 400 + 60 + 40 + 6 = 506$$

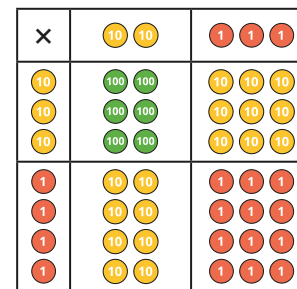
Use Esther's method to work out the multiplications.

32×24

25×32

35×32

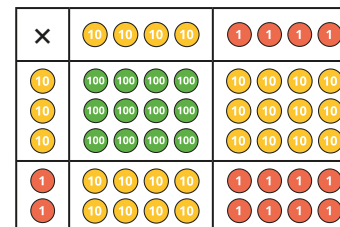
- Aisha uses place value counters and an area model to work out 34×23



$$34 \times 23 = 600 + 90 + 80 + 12 = 782$$

Use Aisha's method to work out 24×32

- Dexter uses place value counters and an area model to work out 44×32



| | | |
|----|-------|-----|
| × | 40 | 4 |
| 30 | 1,200 | 120 |
| 2 | 80 | 8 |

$$44 \times 32 = 1,200 + 80 + 120 + 8 = 1,408$$

Use Dexter's method to work out the multiplications.

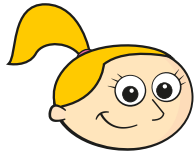
45×42

52×24

34×43

Multiply a 2-digit number by a 2-digit number (area model)

Reasoning and problem solving



To multiply 23 by 57, I just need to calculate 20×50 and 3×7 and then add the products.

Eva's calculation does not include 20×7 and 50×3

What mistake has Eva made?

Explain your answer.



$$42 \times \underline{\quad} = 504$$

Complete the area model to find the missing number.

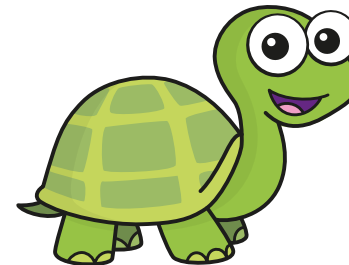
| | | |
|---|--|--|
| × | | |
| | | |
| | | |

12

Mr Trent has a field that measures 53 m long and 25 m wide.

Mrs Lee has a field that measures 52 m long and 26 m wide.

The fields have the same area because the length is 1 m less and the width is 1 m more.



Do you agree with Tiny?

Explain your answer.

No

Multiply a 2-digit number by a 2-digit number

Notes and guidance

In this small step, children progress from the area model to using the formal written method for multiplication.

Encourage children to recognise the links between the area model and the formal method, noting where the subtotals in the formal method match the totals of parts of the area model. This will support children's understanding of each step of the calculation process. A common error when using the formal written method for multiplication is for children to omit the zero placeholder in the ones column when multiplying by the tens digit. Comparing to the area model should make it clear to children why this is needed.

Children can check their answers by rounding to find estimates, for example 42×32 is about $40 \times 30 = 1,200$, so the actual answer should be close to this.

Things to look out for

- Children may omit the zero as a placeholder when multiplying by the tens digit.
- When an exchange is needed in the multiplication steps, children may accidentally also add the exchanged number in the final addition. Crossing out the exchange once it has been used may help to prevent this.

Key questions

- What are you multiplying _____ by first? What are you multiplying _____ by next? Why is this different?
- Why is there a zero in the ones column when multiplying by _____? (for example, when multiplying 14 by 30)
- What do you do after you have multiplied both numbers?
- Where do you write the exchanged ones/tens/hundreds?
- Have you included all the exchanges in your totals?
- How can you use rounding to find an estimate for the answer to the calculation?

Possible sentence stems

- First, I multiply _____ by _____ ones.
Then I multiply _____ by _____ tens.
Finally, I add together _____ and _____

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Multiply a 2-digit number by a 2-digit number

Key learning

- Annie and Tom are working out 32×13

Annie's method

| | | |
|----|-----|----|
| × | 10 | 3 |
| 30 | 300 | 90 |
| 2 | 20 | 6 |

$300 + 90 + 20 + 6 = 416$

Tom's method

| | | | | |
|---|--|---|---|-----------|
| | | | | |
| | | 3 | 2 | |
| × | | 1 | 3 | |
| | | 9 | 6 | (32 × 3) |
| | | 3 | 2 | 0 |
| | | 4 | 1 | 6 |
| | | 1 | | (32 × 10) |

What is the same and what is different about Annie's and Tom's methods?

- Complete the calculation to work out 23×14

| | | | | |
|---|--|---|---|-----------|
| | | | | |
| | | 2 | 3 | |
| × | | 1 | 4 | |
| | | 9 | 2 | (23 × 4) |
| | | 2 | 3 | 0 |
| | | | | (23 × 10) |

Use this method to work out the multiplications.

34×26 58×15 72×35

- Complete the calculation.

| | | | | |
|---|--|---|---|---|
| | | | | |
| | | 4 | 6 | |
| × | | 2 | 7 | |
| | | 3 | 2 | 2 |
| | | 9 | 2 | 0 |

(_____ × _____)
(_____ × _____)

Use this method to work out the multiplications.

27×39 46×55 94×49

- Work out the multiplications.

38×12 39×12 38×11

What is the same and what is different about the answers?
Could you have worked any of them out a different way?

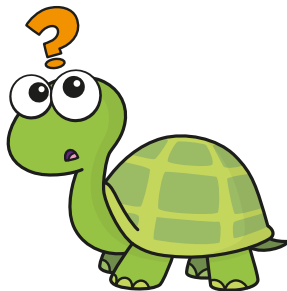
- Mo reads 16 pages of his book every night for 4 weeks. How many pages does he read in total?
Compare methods with a partner.

Multiply a 2-digit number by a 2-digit number

Reasoning and problem solving

Tiny has multiplied 47 by 36

| | | | | | |
|--|---|---|---|---|--|
| | | | | | |
| | | | 4 | 7 | |
| | × | | 3 | 6 | |
| | | 2 | 8 | 2 | |
| | | 1 | 4 | 1 | |
| | | 4 | 2 | 3 | |
| | | 1 | | | |
| | | | | | |
| | | | | | |

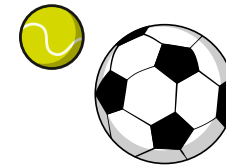


What mistake has Tiny made?
What is the correct answer?

Tiny has forgotten to use zero as a placeholder when multiplying by 3 tens.

1,692

Tennis balls are sold in packs of 34



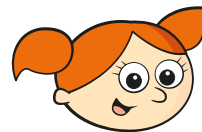
Footballs are sold in packs of 14

A school buys 20 packs of tennis balls and 42 packs of footballs.

How many more tennis balls were bought than footballs?

92

Alex is working out 59×32



$$59 \times 32 = 448$$

Use estimation to show that Alex must be incorrect.

What is the correct answer?

$$60 \times 30 = 1,800$$

1,888

Multiply a 3-digit number by a 2-digit number

Notes and guidance

In this small step, children build on their understanding of multiplying a 2-digit number by a 2-digit number using the formal written method for multiplication and extend it to multiplying a 3-digit number by a 2-digit number.

It is important that children are confident with the previous step before moving on to this one and it may be necessary to refer back to the area model for clarification. Again, ensure that children have an understanding of the role of zero in the ones column when multiplying by the tens digit.

Children use the formal written method for multiplication to solve multi-step problems, including problems from other topics of mathematics such as area.

Things to look out for

- Children may omit the zero as a placeholder when multiplying by the tens digit.
- Children may forget to include exchanges in their calculations.
- If children write the 3-digit number below the 2-digit number in the formal method, they may struggle to work out the answer.

Key questions

- What are you multiplying _____ by first?
What are you multiplying _____ by next?
Why is this different?
- Why is there a zero in the ones column when multiplying by _____? (for example, when multiplying 314 by 30)
- Where do you put the exchanged ones/tens/hundreds?
- What do you need to do to complete the calculation?
- What is the same and what is different about multiplying a 2-digit number by a 2-digit number and multiplying a 3-digit number by a 2-digit number?

Possible sentence stems

- First, I multiply _____ by _____ ones.
Then I multiply _____ by _____ tens.
Finally, I add together _____ and _____

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Multiply a 3-digit number by a 2-digit number

Key learning

- Complete the calculation to work out 123×23

| | | | | | | | |
|--|---|--|---|---|---|---|--|
| | | | | | | | |
| | | | 1 | 2 | 3 | | |
| | × | | | 2 | 3 | | |
| | | | 3 | 6 | 9 | | |
| | | | 2 | 4 | 6 | 0 | |
| | | | | | | | |
| | | | | | | | |

(123 × 3)
(123 × 20)

Use this method to work out the multiplications.

312×13

243×21

202×34

- Complete the calculation to work out 284×37

| | | | | | | | |
|--|---|--|----------------|----------------|----------------|---|--|
| | | | | | | | |
| | | | 2 | 8 | 4 | | |
| | × | | | 3 | 7 | | |
| | | | 1 | 9 ₅ | 8 ₂ | 8 | |
| | | | 8 ₂ | 5 ₁ | 2 | 0 | |
| | | | | | | | |
| | | | | | | | |

(_____ × _____)
(_____ × _____)

Use this method to work out the multiplications.

436×25

537×32

46×291

- Estimate the answers to the multiplications.

637×24

573×28

573×82

Work out the multiplications.

Compare your estimates to your answers.

- A playground is 128 yards by 73 yards.
Work out the area of the playground.
- Tickets for a plane flight cost £246
There are 64 seats on the plane.
How much does it cost to buy all 64 seats for the flight?
- A rugby pitch is 112 m long and 68 m wide.
What is the area of the pitch?
A football pitch is 1 m longer and 1 m narrower than the rugby pitch.
Which pitch has the greater area?

Multiply a 3-digit number by a 2-digit number

Reasoning and problem solving

$$22 \times 111 = 2,442$$

$$23 \times 111 = 2,553$$

$$24 \times 111 = 2,664$$



What do you think the answer to 25×111 will be?

Does this always work?



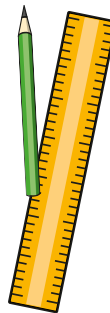
2,775

Pencils are sold in boxes of 64

Rulers are sold in boxes of 46

A school buys 270 boxes of pencils and 720 boxes of rulers.

How many more rulers than pencils does the school buy?



15,840



Tiny has done some calculations.

| | | | | | | |
|--|---|---|---|----------------|----------------|---|
| | | | | | | |
| | | | 9 | 8 | 7 | |
| | × | | | 7 | 6 | |
| | | | 5 | 9 ₅ | 2 ₄ | 2 |
| | | | 6 | 9 ₆ | 0 ₄ | 9 |
| | | 1 | 2 | 8 | 3 | 1 |
| | | | 1 | | 1 | |

| | | | | | | | |
|--|---|--|---|----------------|----------------|----------------|---|
| | | | | | | | |
| | | | 3 | 2 | 4 | | |
| | × | | | 7 | 8 | | |
| | | | | 2 | 5 ₁ | 9 ₃ | 2 |
| | | | 2 | 2 ₁ | 6 ₂ | 8 | 0 |
| | | | 3 | 2 | 7 | 2 | |
| | | | 1 | 1 | | | |

75,012

25,272

What mistakes has Tiny made?

Find the correct answers.



Multiply a 4-digit number by a 2-digit number

Notes and guidance

In this small step, children build on their understanding from the previous two steps to multiply a 4-digit number by a 2-digit number.

Children need to be confident with multiplying 2-digit numbers by both 2- and 3-digit numbers before moving on to this step. As they are now working with greater numbers, it is important that children understand the steps taken when using the long multiplication method. An area model using place value counters could potentially be useful to support children who need it, but the emphasis should be on using the formal written method.

As with the previous steps, children need to understand the role of zero in the ones column when multiplying by the tens.

The main focus of this small step is for children to practise completing multiplications of this sort before moving on to solve problems in the next step.

Things to look out for

- Children may omit the zero as a placeholder when multiplying by the tens digit.
- Children may forget to include exchanges in their calculations.
- If children write the 2-digit number on top when setting up their formal method, they may struggle to complete the calculation.

Key questions

- What are you multiplying _____ by first?
What are you multiplying _____ by next?
Why is this different?
- Why is there a zero in the ones column when multiplying by _____? (for example, when multiplying 2,314 by 30)
- Where do you put the exchanged ones/tens/hundreds/thousands?
- What do you do to complete the calculation?

Possible sentence stems

- First, I multiply _____ by _____ ones.
Then I multiply _____ by _____ tens.
Finally, I add together _____ and _____

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Multiply a 4-digit number by a 2-digit number

Key learning

- Complete the calculations.

| | | | | | | |
|---|--|---|---|---|---|---|
| | | | | | | |
| | | 3 | 2 | 4 | 2 | |
| × | | | | 2 | 1 | |
| | | 3 | 2 | 4 | 2 | |
| | | 6 | 4 | 8 | 4 | 0 |
| | | | | | | |
| | | | | | | |
| | | | | | | |

(3,242 × _____)

(3,242 × _____)

| | | | | | | |
|---|--|---|----------------|----------------|----------------|---|
| | | | | | | |
| | | 3 | 2 | 4 | 2 | |
| × | | | | 2 | 6 | |
| | | 1 | 9 ₁ | 4 ₂ | 5 ₁ | 2 |
| | | 6 | 4 | 8 | 4 | 0 |
| | | | | | | |
| | | | | | | |
| | | | | | | |

(3,242 × _____)

(3,242 × _____)

| | | | | | | |
|---|--|---|---|---|---|--|
| | | | | | | |
| | | 4 | 2 | 3 | 6 | |
| × | | | | 5 | 2 | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

(_____ × _____)

(_____ × _____)

| | | | | | | |
|---|--|---|---|---|---|--|
| | | | | | | |
| | | 3 | 4 | 7 | 2 | |
| × | | | | 6 | 4 | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

(_____ × _____)

(_____ × _____)

- Find the product of 3,064 and 43

- Estimate the answers to the multiplications.

3,282 × 32

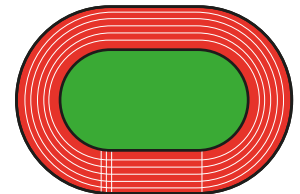
7,132 × 21

9,708 × 38

Work out the multiplications.

How close were your estimates to the actual answers?

- A race is 5,407 m long.
36 runners complete the race.
What is the combined total distance run?



- Write <, > or = to compare the calculations.

4,458 × 56 ○ 4,523 × 54

4,458 × 55 ○ 4,523 × 54

4,458 × 55 ○ 4,522 × 54

Did you need to work out the calculations each time?

Multiply a 4-digit number by a 2-digit number

Reasoning and problem solving

| | | | | | | | |
|--|---|---|---|---|---|---|--|
| | | | | | | | |
| | | 2 | 5 | 3 | 4 | | |
| | × | | | 2 | 3 | | |
| | | 7 | 5 | 9 | 2 | | |
| | | 5 | 0 | 6 | 8 | | |
| | | 1 | 2 | 6 | 6 | 0 | |
| | | | 1 | 1 | | | |

58,282

What are the mistakes in this calculation?

Work out the correct answer.



Teddy has spilt some paint on this multiplication.

| | | | | | | | |
|--|---|---|---|---|---|---|---|
| | | | | | | | |
| | | 2 | 6 | 9 | | | |
| | × | | 2 | | | | |
| | | 2 | 2 | 9 | 5 | 7 | 2 |
| | | 5 | 7 | 3 | | 0 | |
| | | 0 | 3 | 3 | 2 | | |
| | | 1 | 1 | 1 | | | |

All the missing digits are 8

What are the missing digits?



Tiny thinks these multiplications will have the same answer.

$1,342 \times 23$

$1,341 \times 24$

No

Why might Tiny think this?

Is Tiny correct?



0 2 3 5 6 9

Arrange the digits in the multiplication to make the greatest possible product.

□ □ □ □ × □ □

6,520 × 93

Solve problems with multiplication

Notes and guidance

In this small step, children apply their knowledge of multiplication to solve problems.

Children practise both the formal written method for multiplication and the use of efficient mental strategies. It is important that children explore a variety of methods to solve multiplication problems and discuss which is the most efficient. They may refer to known facts to help them derive unknown facts. For example, to calculate $9,999 \times 6$, they can calculate $10,000 \times 6$ and then subtract 1 lot of 6

Building on their learning from Year 4 (where they multiplied three numbers), children should use their knowledge of multiplication being commutative to multiply the numbers in any order, depending on which is the most efficient.

Things to look out for

- Children may not identify the correct order in which to complete the different calculations.
- Children may become over-reliant on the formal multiplication method even when there is a more efficient mental strategy.
- If children are not confident with their times-tables, they may find it harder to derive unknown facts.

Key questions

- What operation do you need to do? How do you know?
- Why can you multiply the numbers in any order?
- What strategy can you use to solve this problem?
- How do the words in the problem tell you what to do?
- Is there a more efficient method?
- What calculation do you need to do? How do you know?
- Could you have worked it out a different way?

Possible sentence stems

- To calculate $_____ \times 24$, I can do $_____ \times _____ \times _____$
- To calculate $9,999 \times _____$, I can do $10,000 \times _____ - _____$
- The most efficient strategy to calculate $_____ \times _____$ is ...

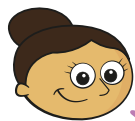
National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers

Solve problems with multiplication

Key learning

- Dora and Jack have worked out 46×99



Dora

I used the long multiplication method to work out 46×99 and got 4,554

I calculated 46×100 , which is 4,600, and then subtracted 1 lot of 46 to get 4,554



Jack

Explain why both methods work.

Which method do you prefer? Why?

Use your preferred method to work out the multiplications.

$$24 \times 102$$

$$324 \times 99$$

$$198 \times 52$$

- Without calculating, write $<$, $>$ or $=$ to compare the calculations.

$$2,470 \times 83 \quad \bigcirc \quad 247 \times 830$$

$$4,642 \times 24 \quad \bigcirc \quad 4,641 \times 25$$

Explain your reasoning.

- 30 children in Class 5 are raising money for charity. 12 children raise £85 each, 8 children raise £240 each and the rest raise £100 each. How much have the children raised altogether?

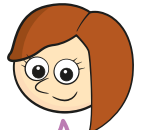
- Tommy and Rosie have worked out 284×24



Tommy

I multiplied 284 by 3, which is 852, and then multiplied 852 by 8 to get 6,816

Rosie



I multiplied 284 by 4, and then multiplied the answer by 6 to get 6,816

Explain why both methods work.

Which method do you prefer? Why?

Work out the multiplications.

$$25 \times 286$$

$$647 \times 18$$

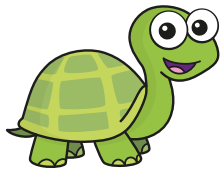
$$539 \times 32$$

- A machine makes 2,346 bags every day. How many bags will it make in 3 weeks?
- A pilot flies a plane 1,268 miles every day in August and September. How many miles does the pilot fly in total?

Solve problems with multiplication

Reasoning and problem solving

Tiny is working out $6,999 \times 99$



I can do
 $6,999 \times 100$
and then
subtract 1

Is Tiny correct?

Explain your answer.



No

FLIGHTS TO AUSTRALIA



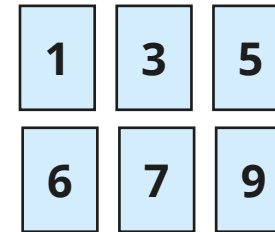
| | |
|-------------|--------|
| Economy | £2,464 |
| First Class | £4,502 |

On a flight to Australia, there are 46 economy seats and 18 first class seats.

Tickets for all the seats are sold.

How much money does the airline receive from ticket sales?

£194,380



Arrange the digit cards in the multiplication.



What is the greatest product that can be made?

What is the smallest product that can be made?

What is the difference between the greatest and smallest product?

$$9,531 \times 76 = 724,356$$

$$3,679 \times 15 = 55,185$$

$$669,171$$

Short division

Notes and guidance

Building on informal methods used in Years 3 and 4, this small step introduces children to the formal written method of short division.

The formal calculation is shown alongside familiar models, in particular part-whole models, place value counters and place value charts. In this way, the structure of short division becomes clear, enabling children to see the relationship between the model and the formal written method.

First, children use the formal method to divide a 2-digit number by a 1-digit number, initially without an exchange and then with an exchange. They then divide a 3-digit number by a 1-digit number, again without and then with an exchange. Dividing 4-digit numbers is covered in the next step, with calculations involving remainders following later in the block.

Things to look out for

- Children may need support to understand the process of exchanging in this new format.
- Children may work from right to left, as with addition, subtraction and multiplication.
- When dividing numbers that include zeros as placeholders, children may make errors with place value.

Key questions

- Which digit do you divide first?
- How many groups of hundreds/tens/ones are there?
- How can you set out the division using the formal written method?
- When using short division, do you start from the left or the right?
- When do you need to make an exchange?

Possible sentence stems

- _____ hundreds divided by _____ is equal to _____ hundreds with a remainder of _____
- Exchange the remainder, then _____ tens divided by _____ is equal to _____ tens with a remainder of _____
- Exchange the remainder, then _____ ones divided by _____ is equal to _____ ones.

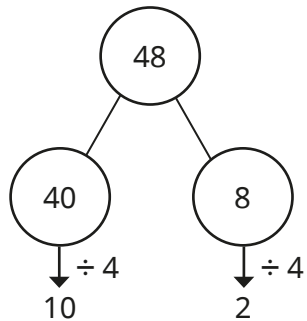
National Curriculum links

- Divide up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context

Short division

Key learning

- What is the same and what is different about the two methods for dividing 48 by 4?



| | | | | |
|--|---|---|---|--|
| | | | | |
| | | 1 | 2 | |
| | 4 | 4 | 8 | |
| | | | | |

$10 + 2 = 12$, so $48 \div 4 = 12$

- Complete the sentences to describe how the place value chart shows $39 \div 3$

| T | O |
|----------|-------|
| 10 10 10 | 1 1 1 |
| | 1 1 1 |
| | 1 1 1 |

| | | | | |
|--|---|---|---|--|
| | | | | |
| | | 1 | 3 | |
| | 3 | 3 | 9 | |
| | | | | |

There is _____ group of 3 tens.

There are _____ groups of 3 ones.

$39 \div 3 = \underline{\quad} + \underline{\quad}$
 $= \underline{\quad}$

- Circle groups of 3 counters to calculate $963 \div 3$

Complete the short division.

| H | T | O |
|-------------|----------|-------|
| 100 100 100 | 10 10 10 | 1 1 1 |
| 100 100 100 | 10 10 10 | |
| 100 100 100 | | |

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

- Sam uses a place value chart and counters to work out $605 \div 5$

| H | T | O |
|---------------------|----------------|-----------|
| 100 100 100 100 100 | 10 10 10 10 10 | 1 1 1 1 1 |
| 100 | 10 10 10 10 10 | |

| | | | | |
|--|---|---|---|---|
| | | | | |
| | | 1 | 2 | 1 |
| | 5 | 6 | 0 | 5 |
| | | | | |

Sam exchanges the remaining hundred counter for 10 ten counters.

Use Sam's method to work out the divisions.

$426 \div 3$

$786 \div 6$

$532 \div 4$

- Use short division to work out the divisions.

$844 \div 4$

$684 \div 6$

$540 \div 4$

$804 \div 3$

Short division

Reasoning and problem solving

Find the missing numbers.



| | | | | | |
|---|---|---|---|--|--|
| | | | | | |
| | 1 | | 6 | | |
| 3 | | 9 | 8 | | |

| | | | | | |
|---|---|---|---|--|--|
| | | | | | |
| | 1 | 6 | 6 | | |
| 3 | 4 | 9 | 8 | | |

Huan is calculating $725 \div 5$

| | | | | | |
|---|---|---|---|--|--|
| | | | | | |
| | 1 | 0 | 5 | | |
| 5 | 7 | 2 | 5 | | |

What mistake has Huan made?

In the first step of the division, Huan has not exchanged the 2 remaining hundreds for tens.

Which calculation is the odd one out?

$363 \div 3$

$824 \div 2$

$524 \div 4$

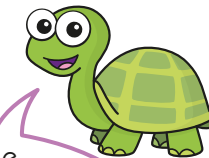
$777 \div 7$

multiple possible answers, e.g. $524 \div 4$, because this is the only calculation that requires an exchange

Explain your answer.



| | | |
|---|---|-----|
| | 2 | 1 |
| 4 | 8 | 0 4 |



To divide 804 by 4, I worked out 8 divided by 4 is 2 and 4 divided by 4 is 1, so $804 \div 4 = 21$

What mistake has Tiny made?

What is the correct answer?

Tiny has not included the zero as a placeholder.

201

Divide a 4-digit number by a 1-digit number

Notes and guidance

Following the introduction of formal short division in the previous step, in this small step children move on to dividing a 4-digit number by a 1-digit number.

Place value counters continue to be used to represent the calculations alongside the formal written method, so that children can visualise how one relates to the other. In particular, place value counters in place value charts help children to make sense of the steps that they are taking and how this relates to the context of the question.

Children begin with divisions that have no exchanges and then progress to those with exchanges. Divisions with remainders are covered in the next step.

Things to look out for

- Children may need support to understand the process of exchanging in divisions.
- Children may work from right to left, as with addition, subtraction and multiplication.
- When dividing numbers that include zeros as placeholders, children may make errors with place value.

Key questions

- How would you set out a division using the formal written method?
- Which digit do you divide first?
- When using short division, do you start from the left or the right?
- What do you do if the number you are dividing by does not divide exactly into the first digit?
- When do you need to make an exchange?

Possible sentence stems

- To use the formal method of division, I start with the digit on the _____ and work from _____ to _____
- There are _____ groups of _____ thousands/hundreds/tens/ones in _____ thousands/hundreds/tens/ones.

National Curriculum links

- Divide up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context

Divide a 4-digit number by a 1-digit number

Key learning

- Use the place value chart to work out $9,639 \div 3$

| Th | H | T | O |
|-------------------|-------------|----------|-------|
| 1,000 1,000 1,000 | 100 100 100 | 10 10 10 | 1 1 1 |
| 1,000 1,000 1,000 | 100 100 100 | | 1 1 1 |
| 1,000 1,000 1,000 | | | 1 1 1 |

| | | | | | |
|---|---|---|---|---|--|
| | | | | | |
| | | | | | |
| 3 | 9 | 6 | 3 | 9 | |
| | | | | | |
| | | | | | |

- Ron has worked out $4,892 \div 4$ using place value counters and short division.

| Th | H | T | O |
|-------------|---------|-------|-----|
| 1,000 1,000 | 100 100 | 10 10 | 1 1 |
| 1,000 1,000 | 100 100 | 10 10 | 1 1 |
| | 100 100 | 10 10 | 1 1 |
| | 100 100 | 10 10 | 1 1 |
| | | 10 | 1 1 |
| | | | 1 1 |

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | 1 | 2 | 2 | 3 |
| 4 | 4 | 8 | 9 | 2 | |
| | | | | | |

Use place value counters and short division to work out the divisions.

| | | |
|----------------|----------------|----------------|
| $6,610 \div 5$ | $2,472 \div 3$ | $9,360 \div 4$ |
|----------------|----------------|----------------|

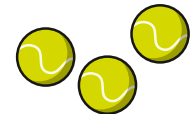
- Complete the divisions.

| | | | | | |
|---|---|---|---|---|--|
| | | | | | |
| | | | | | |
| 8 | 8 | 9 | 7 | 6 | |
| | | | | | |
| | | | | | |

| | | | | | |
|---|---|---|---|---|--|
| | | | | | |
| | | | | | |
| 3 | 5 | 0 | 2 | 2 | |
| | | | | | |
| | | | | | |

| | | | | | |
|---|---|---|---|---|--|
| | | | | | |
| | | | | | |
| 7 | 9 | 1 | 2 | 8 | |
| | | | | | |
| | | | | | |

- 9,632 tennis balls are packed into boxes of 8
How many boxes will be needed?



- A school raises £8,934 for charity.
The money is shared equally between three charities.
How much money does each charity receive?

- A plane travels the same distance every day.
Altogether the plane travels 6,363 miles in a week.
How far does it travel each day?



- 5,427 marbles are put into bags with 9 marbles in each bag.
The bags are shared equally between three boxes.
How many bags of marbles are in each box?

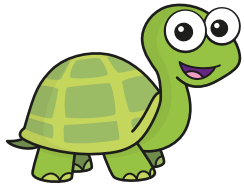


Divide a 4-digit number by a 1-digit number

Reasoning and problem solving

Tiny is working out $2,240 \div 7$

This cannot be done because 7 is greater than each of the digits in the number.



Do you agree with Tiny?

Explain your answer.

No

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| | | 3 | 1 | 0 | 1 |
| 3 | 9 | 4 | 1 | 4 | |
| | | | | | |

3,138

What mistakes have been made?

What is the correct answer?

Fill in the missing numbers.

| | | | | | |
|--|---|---|---|---|--|
| | | | | | |
| | 1 | 3 | | 4 | |
| | 7 | 8 | 2 | 2 | |
| | | | | | |

| | | | | | |
|---|---|---|---|---|--|
| | | | | | |
| | 1 | 3 | 0 | 4 | |
| 6 | 7 | 8 | 2 | 4 | |
| | | | | | |

Write $<$, $>$ or $=$ to compare the calculations.

$3,495 \div 5$ $3,495 \div 3$

$8,064 \div 7$ $9,198 \div 7$

$4,244 \div 4$ $8,488 \div 8$

$<$
 $<$
 $=$

Did you need to work out all the divisions?

Divide with remainders

Notes and guidance

In previous years, children have looked at division with remainders informally. In this small step, they move on to formal calculations that result in a remainder.

The formal written method for short division continues to be used alongside familiar models. Children use place value charts and counters so that they associate the remainder with the amount “left over”. The progression of examples is carefully chosen to focus children’s attention on the link between the remainder and the number being divided by. They should generalise that a remainder must be less than the number being divided by. Remainders are represented in the calculation as r_1 , r_2 and so on.

In this step, the focus is on completing and understanding the calculation procedure. Making decisions about the remainder based on the context of the question is covered in Step 11

Things to look out for

- Children may make the incorrect generalisation that the remainder is always 1
- Errors in calculation may lead to children writing remainders that are greater than the number being divided by.

Key questions

- What does “remainder” mean?
- How can you use your times-tables to know if a division by $2/5$ will have a remainder? What other divisibility rules do you know?
- What do you notice about the size of the remainders compared to the number being divided by?
- What is the greatest possible remainder you can get when dividing by _____?
- How do you know this answer is incorrect, just by looking at the size of the remainder?

Possible sentence stems

- _____ ones divided by _____ = _____ ones remainder _____
- When dividing by _____, the greatest possible remainder is _____

National Curriculum links

- Divide up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context

Divide with remainders

Key learning

- 13 sweets are shared equally between 4 people.

How many sweets are left over?

- Mo wants to put 27 pencils into pots of 4

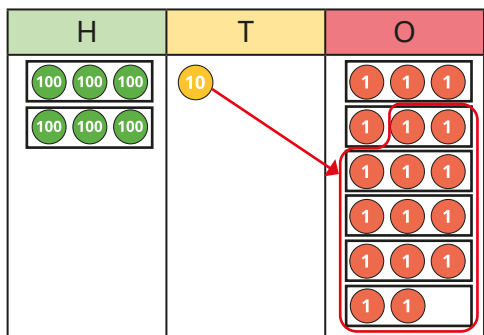
How many pots of 4 pencils can he make?

How many pencils are left over?

Complete the division sentence.

_____ ÷ _____ = _____ r _____

- Nijah works out $617 \div 3$ using place value counters and a place value chart, and then writes the formal method.



| | | | | | |
|---|--|---|---|---|----|
| | | | | | |
| | | 2 | 0 | 5 | r2 |
| 3 | | 6 | 1 | 7 | |
| | | | | | |

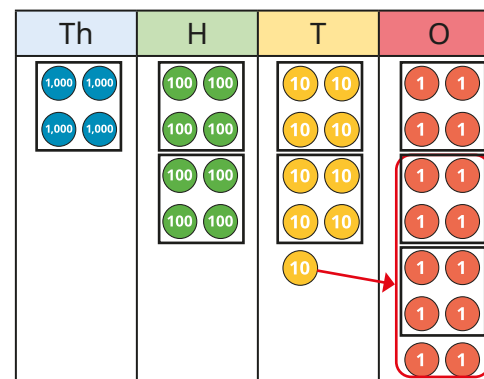
Use Nijah's method to work out the divisions.

$613 \div 5$

$473 \div 3$

$963 \div 4$

- Scott is working out $4,894 \div 4$



| | | | | | |
|---|--|---|---|---|------|
| | | | | | |
| | | 1 | 2 | 2 | 3 r2 |
| 4 | | 4 | 8 | 9 | 4 |
| | | | | | |

Use Scott's method to work out the divisions.

$6,613 \div 5$

$2,471 \div 3$

$9,363 \div 4$

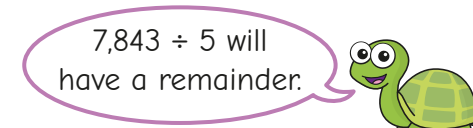
- In a factory, muffins are packed into boxes of 6

One day, the factory makes 5,625 muffins.



How many muffins will not be boxed?

-



Explain how Tiny knows this.

Divide with remainders

Reasoning and problem solving

Work out the divisions.

$$36 \div 3 \quad 37 \div 3 \quad 38 \div 3 \quad 39 \div 3$$

What do you notice about the remainders?

What does this tell you about remainders?



12, 12 r1, 12 r2, 13

Find the missing numbers.



| | | | | | | | |
|---|---|---|---|---|----|--|--|
| | | | | | | | |
| | 0 | | 6 | 1 | r4 | | |
| 5 | | 4 | 3 | 0 | | | |
| | | | | | | | |

| | | | | | | | |
|---|---|---|---|---|----|--|--|
| | | | | | | | |
| | 0 | 8 | 6 | 1 | r4 | | |
| 5 | 4 | 3 | 0 | 9 | | | |
| | | | | | | | |

Amir is thinking of a 3-digit number that is less than 500



When my number is divided by 9, the remainder is 3. When my number is divided by 2, the remainder is 1. When my number is divided by 5, the remainder is 4

What could Amir's number be?

129, 219, 309, 399, 489

Is the statement always true, sometimes true or never true?

When a 3-digit number made of consecutive, descending digits is divided by the next digit, the remainder is 1

For example, $765 \div 4 = 191 \text{ r}1$

sometimes true

Explain your answer.



Efficient division

Notes and guidance

So far in this block, children have divided numbers with up to four digits in a range of contexts, using various methods. They have used informal methods to understand the structure of division and the formal written method to promote efficiency.

In this small step, children consolidate their knowledge and understanding of division and begin to make decisions regarding the most efficient or appropriate methods to use in a range of contexts. They begin by looking at informal methods, such as partitioning, using known facts, factor pairs and number lines, and then compare these to the formal written method. They make decisions about which method they prefer or which would be more efficient for a given problem.

Things to look out for

- Children may become over-reliant on the formal written method instead of considering alternative approaches that may be more efficient.
- Children may partition the number being divided by, rather than using factors to break up the calculation, for example $12 \div 6 = 12 \div 4 \div 2$ rather than $12 \div 6 = 12 \div 2 \div 3$

Key questions

- Which method do you find the easiest?
- Which method do you find the most efficient?
- How would you explain how this method works?
- What is the most efficient way to divide _____ by _____?
- What happens if you double one factor and halve the other?
- How can you use factor pairs to help you?
- How can you divide multiples of ten?

Possible sentence stems

- To divide by 4, I can divide by _____ and then divide the result by _____
- To divide by 8, I can divide by 2 _____ times.
- To divide by 6, I can divide by _____ and then divide the result by _____

National Curriculum links

- Divide up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context

Efficient division

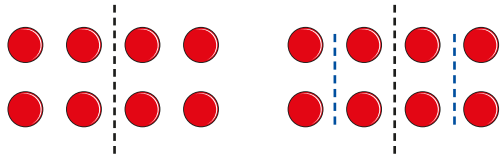
Key learning

- Complete the divisions.

▶ $300 \div 1 = \underline{\quad}$ ▶ $300 \div 10 = \underline{\quad}$ ▶ $300 \div 100 = \underline{\quad}$

What do you notice?

- The array shows that $8 \div 4 = 8 \div 2 \div 2$



Make your own arrays to show these divisions.

$16 \div 4 = 16 \div 2 \div 2$

$32 \div 8 = 32 \div 2 \div 2 \div 2$

- Mo uses factors to work out $810 \div 6$

Factors of 6 are 2 and 3:

| | | |
|--------------------|----|--------------------|
| $810 \div 2 = 405$ | or | $810 \div 3 = 270$ |
| $405 \div 3 = 135$ | | $270 \div 2 = 135$ |

So $810 \div 6 = 135$

Use Mo's method to work out the divisions.

$126 \div 6$

$960 \div 6$

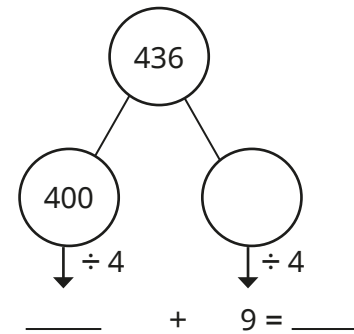
$1,392 \div 6$

$2,052 \div 6$

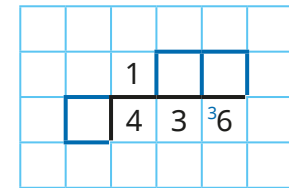
- Here are four different ways of working out $436 \div 4$

Complete the calculation in each method.

Method 1: Partitioning



Method 2: Short division

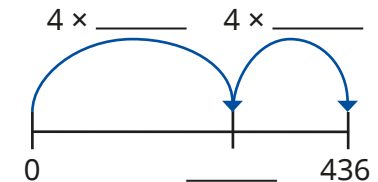


Method 3: Half and half again

$436 \div \underline{\quad} = 218$

$218 \div 2 = \underline{\quad}$

Method 4: Finding groups of 4 along a number line



Which method would you use to work out these divisions?

$220 \div 4$

$648 \div 6$

$805 \div 7$

$114 \div 6$

Use your chosen method to work out each division.

Efficient division

Reasoning and problem solving

All numbers that end in 2 are divisible by 2, all numbers that end in 5 are divisible by 5 and all numbers that end in 0 are divisible by 10. So, all numbers that end in 3 will be divisible by 3

No

Is Whitney correct?
Explain your answer.

I know that $12 \div 4 = 3$ and $120 \div 4 = 30$, so $120 \div 40 = 300$

No

Do you agree with Ron?
Explain your answer.

Dexter, Eva and Annie each choose one of the number cards.

976 100,000 4,968

They divide their number by 8

I partitioned my number into 800, 160 and 16, then divided each part by 8

I used short division and had just one exchange.

I halved my number, then halved it again, then halved it again.

Which number card did each child choose?
Which method would you use to divide each number?

Dexter: 976 Eva: 4,968 Annie: 100,000

Solve problems with multiplication and division

Notes and guidance

In this small step, children apply their knowledge of multiplication and division to solve problems. The main focus of the step is on giving children the opportunity to choose which operation is needed in order to answer a particular problem, and then to solve the problem. Pictorial representations, such as bar models, can support children's understanding.

Children also develop their understanding of the remainder when performing a division in context. For example, if pencils come in packs of 4 and a class needs 30 pencils, how many packs are needed? Children may recognise that they need to divide 30 by 4, which is equal to 7 remainder 2. However, in order to answer this question correctly, they need to be aware that 8 packs are needed. In a different context, 7 remainder 2 may mean only 7 full packs can be made.

Things to look out for

- Children may be unsure which operation is needed to solve a problem.
- Children may be able to divide using a procedure, but lack understanding of the remainder in a particular context.

Key questions

- What calculation do you need to do? How do you know?
- What does the remainder represent in this problem?
- Do you need more or fewer boxes/bags? What does the remainder mean here?
- How do you know if you need to add an extra box/bag?
- How many boxes can be filled? How many boxes do you need?
- Which operation is needed?

Possible sentence stems

- _____ ÷ _____ = _____ remainder _____
- There are _____ left over, so _____ are needed altogether.

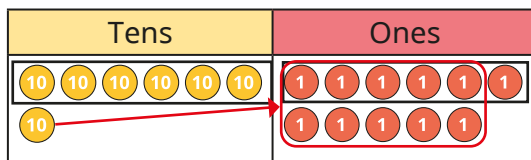
National Curriculum links

- Divide up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Solve problems with multiplication and division

Key learning

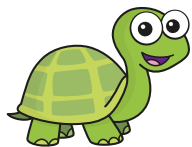
- A minibus can seat 6 people.
71 people are going on a trip.
How many minibuses are needed?
Complete the sentences.



| | | | | |
|--|---|---|----|----|
| | | | | |
| | | 1 | 1 | r5 |
| | 6 | 7 | 11 | |
| | | | | |

There are _____ groups of 6 people.
There are _____ people left over.
_____ minibuses are needed.

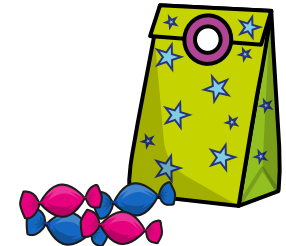
- There are 357 people at a wedding.
They are sitting at tables in groups of 8
Tiny works out how many tables are needed.



$$375 \div 8 = 44 \text{ r}5$$

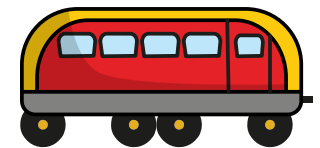
Explain why 45 tables are needed.

- Dani is filling party bags.
Each party bag has 7 sweets in it.
Dani has 349 sweets altogether.
How many party bags can she fill?



- A hockey pitch is 91 m long and 55 m wide.
What is the area of the pitch?
The area of a field is 25,000 m²
How many hockey pitches might fit in it?
How do you know what calculation to do?

- A train has 14 carriages.
Each carriage can carry 42 people.
512 people have reserved a seat.
How many unreserved seats are there?



- A car park has 147 rows of 18 spaces.
110 rows are full and the rest are empty.
How many spaces are empty?

Solve problems with multiplication and division

Reasoning and problem solving

Textbooks come in packs of 6
A school needs 4,607 textbooks.
How many packs are needed?

$$4,607 \div 6 = 767 \text{ r}5$$



Sam

$4,607 \div 6 = 767 \text{ r}5$,
so the school needs
767 packs.



Jack

$4,607 \div 6 = 767 \text{ r}5$,
so the school needs
768 packs.



Teddy

Who do you agree with?
Explain your answer.

Teddy

767 r5 means that
there are 767 packs
of 6 textbooks
with 5 textbooks
left over. So for
the school to have
enough textbooks,
they need to order
768 packs.

Beads come in packs of 8



Scott uses 12 beads to make a bracelet.

He makes 33 bracelets.

How many packs of beads does
he need?

50

pack A



pack B



Mrs Rose needs to buy some
crayons. She orders 13 of pack A
and 22 of pack B.

She puts the crayons into pots, with 8
crayons in each pot.

How many pots does she need?

Compare methods with a partner.



41

Spring Block 2

Fractions B

Small steps

Step 1

Multiply a unit fraction by an integer

Step 2

Multiply a non-unit fraction by an integer

Step 3

Multiply a mixed number by an integer

Step 4

Calculate a fraction of a quantity

Step 5

Fraction of an amount

Step 6

Find the whole

Step 7

Use fractions as operators



Multiply a unit fraction by an integer

Notes and guidance

In this small step, children encounter multiplication number sentences with fractions, multiplying unit fractions by an integer. Make links to multiplication as repeated addition: if children know that $\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$, this will link back to previous learning and avoid the common misconception of multiplying both the numerator and the denominator by the integer.

Bar models are a useful representation and can show the calculations in multiple or single bars. When answers are greater than 1, encourage children to write their answers as a mixed number. They may also find a number line useful.

This learning is built upon in the next few steps, when children multiply non-unit fractions and mixed numbers.

Things to look out for

- Children may think that when multiplying, the answer is always greater than both of the numbers. For example, they may think the result of $3 \times \frac{1}{10}$ must be greater than 3
- Children may multiply both the numerator and the denominator by the integer, and not recognise that this is the process for finding equivalent fractions, not for multiplying fractions by integers.

Key questions

- How can you write this multiplication as a repeated addition? How does this help you to work it out?
- How can you represent this question as a bar model?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What is your answer as a mixed number? What is it as an improper fraction?
- What happens if the integer you are multiplying by is the same as the denominator? Does this always happen?

Possible sentence stems

- $\frac{1}{\square} \times \text{_____} = \frac{1}{\square} + \dots + \frac{1}{\square}$
- To multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.

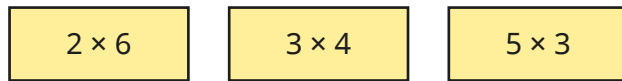
National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Multiply a unit fraction by an integer

Key learning

- Write the multiplications as repeated additions.



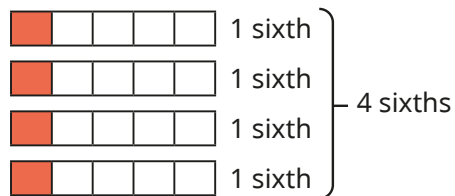
How would you write $\frac{1}{4} \times 3$ as a repeated addition?

- Write the multiplications as repeated additions.

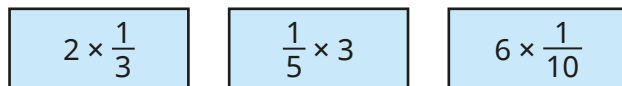


What do you notice?

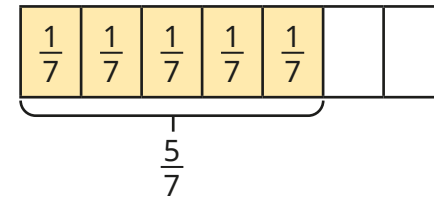
- Ron uses bar models to work out $\frac{1}{6} \times 4 = \frac{4}{6}$



Use Ron's method to work out the multiplications.



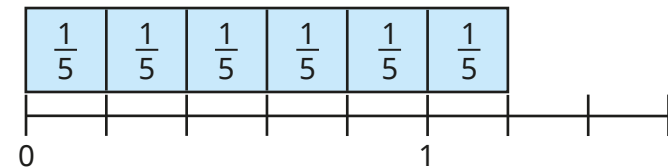
- Alex uses a bar model to work out $5 \times \frac{1}{7} = \frac{5}{7}$



Use Alex's method to work out the multiplications.

- $3 \times \frac{1}{7}$
- $4 \times \frac{1}{10}$
- $7 \times \frac{1}{8}$
- $\frac{1}{10} \times 3$

- Filip uses a number line to work out $\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}$



Use Filip's method to work out the multiplications.

- $\frac{1}{6} \times 7$
- $\frac{1}{3} \times 5$
- $5 \times \frac{1}{5}$
- $9 \times \frac{1}{4}$

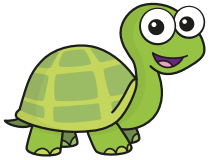
- Complete the multiplications.
Give your answers as mixed numbers.

- $6 \times \frac{1}{4}$
- $\frac{1}{3} \times 4$
- $10 \times \frac{1}{8}$
- $\frac{1}{10} \times 19$

Multiply a unit fraction by an integer

Reasoning and problem solving

Tiny is multiplying a unit fraction by an integer.



$$\frac{1}{5} \times 5 = \frac{5}{25}$$

Explain Tiny's mistake.

Tiny has multiplied the denominator as well as the numerator.

Is the statement always true, sometimes true or never true?

When you multiply a unit fraction by the same number as its denominator, the answer will be 1 whole.

Explain your answer.

always true



I am thinking of a unit fraction.

When I multiply it by 4, it is equivalent to $\frac{1}{2}$

When I multiply it by 2, it is equivalent to $\frac{1}{4}$

What is Mo's fraction?

What does Mo need to multiply his fraction by so that his answer is equivalent to $\frac{3}{4}$?

Write a similar question for a partner to solve.

$\frac{1}{8}$

6

Multiply a non-unit fraction by an integer

Notes and guidance

In this small step, children build on the previous step to multiply non-unit fractions by integers.

As in the previous step, children make the link between multiplication and repeated addition, and use bar models and number lines to support calculations. However, they should become more fluent and recognise the generalisation that they need to multiply the numerator by the integer and leave the denominator the same.

Children need to be able to convert improper fractions to mixed numbers and could use number lines or other representations to help.

In the next small step, children combine their learning from the first two steps to multiply mixed numbers by integers.

Things to look out for

- Children may think that when multiplying, the answer is always greater than both of the numbers. For example, they may think the result of $3 \times \frac{3}{10}$ must be greater than 3
- Children need to be confident in converting between improper fractions and mixed numbers.

Key questions

- How can you write this multiplication as a repeated addition?
- How can you represent this multiplication as a bar model?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What is your answer as a mixed number? What is it as an improper fraction?
- How do you know that $\frac{3}{5} \times 2 = \frac{6}{10}$ can not be correct?

Possible sentence stems

- $\frac{\square}{\square} \times \text{_____} = \frac{\square}{\square} + \dots + \frac{\square}{\square}$
- To multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

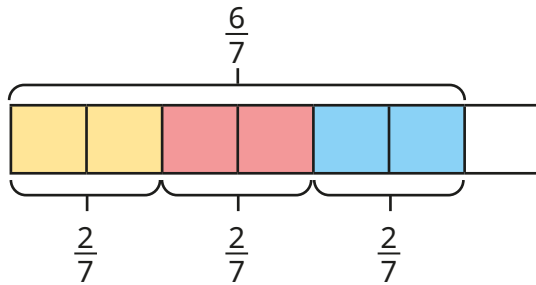
Multiply a non-unit fraction by an integer

Key learning

- Write the multiplications as repeated additions.

| | | | |
|------------------------|-------------------------|------------------------|-------------------------|
| $2 \times \frac{3}{8}$ | $\frac{3}{10} \times 3$ | $\frac{2}{9} \times 4$ | $5 \times \frac{3}{19}$ |
|------------------------|-------------------------|------------------------|-------------------------|

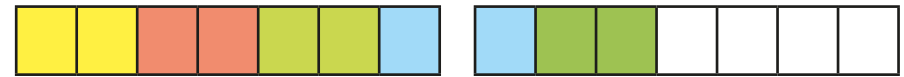
- Brett uses a bar model to work out $3 \times \frac{2}{7} = \frac{6}{7}$



Use Brett's method to work out the multiplications.

| | | | |
|-------------------------|-------------------------|-------------------------|------------------------|
| $2 \times \frac{3}{8}$ | $4 \times \frac{2}{11}$ | $\frac{3}{10} \times 3$ | $\frac{3}{8} \times 2$ |
| $3 \times \frac{5}{19}$ | $\frac{3}{10} \times 2$ | $\frac{2}{13} \times 5$ | $4 \times \frac{2}{9}$ |

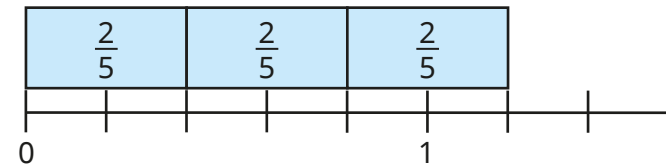
- Dani uses bar models to work out $\frac{2}{7} \times 5 = \frac{10}{7} = 1\frac{3}{7}$



Use Dani's method to work out the multiplications.

▶ $\frac{2}{7} \times 6$ ▶ $\frac{3}{7} \times 5$ ▶ $3 \times \frac{4}{7}$

- Huan uses a number line to help work out $\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}$



Use Huan's method to work out the multiplications.

▶ $\frac{2}{5} \times 4$ ▶ $2 \times \frac{3}{5}$ ▶ $\frac{3}{10} \times 5$ ▶ $\frac{2}{9} \times 9$

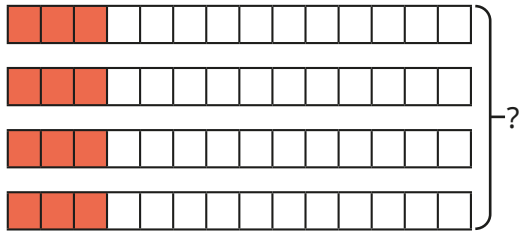
- Complete the multiplications.
Give your answers as mixed numbers.

▶ $\frac{3}{13} \times 5$ ▶ $6 \times \frac{5}{7}$ ▶ $\frac{6}{11} \times 9$ ▶ $8 \times \frac{7}{12}$

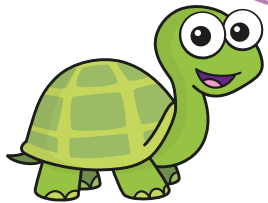
Multiply a non-unit fraction by an integer

Reasoning and problem solving

Tiny has worked out $4 \times \frac{3}{14}$



From the bar model, I can see that $4 \times \frac{3}{14} = \frac{12}{56}$



Do you agree with Tiny?
Explain your answer.

No

The bar model shows 4 lots of $\frac{3}{14}$

$$\frac{3}{14} + \frac{3}{14} + \frac{3}{14} + \frac{3}{14} = \frac{12}{14}$$

Work out the calculations.

$$\frac{3}{4} \times 3$$

$$4 \times \frac{4}{5}$$

$$\frac{5}{6} \times 5$$

What patterns do you notice?

What comes next in the sequence?

$$2\frac{1}{4}, 3\frac{1}{5}, 4\frac{1}{6}$$

$$6 \times \frac{6}{7} = 5\frac{1}{7}$$

Use the digit cards to complete the multiplication.

You can use a card once only in each multiplication.



$$\square \times \frac{\square}{\square} = \frac{\square}{\square}$$

Is there more than one possible answer?

Are there any of the cards you cannot use?

multiple possible answers, e.g.

$$2 \times \frac{1}{3} = \frac{4}{6}$$

Multiply a mixed number by an integer

Notes and guidance

In this small step, children build on their learning from the first two steps to multiply mixed numbers by integers. Children need to be secure in their understanding of multiplying proper fractions by integers before adding the extra challenge of multiplying mixed numbers.

Children explore a range of methods to complete the calculations and discuss the efficiency of each. To build understanding, initially calculations should not involve converting improper fractions to mixed numbers. Once children are secure in using the methods, they can explore questions where in the answer, the fractional part of the calculation is greater than 1 and needs converting to a mixed number before combining the totals.

Things to look out for

- Children may write their answer as a whole number and an improper fraction rather than a mixed number.
- Children may use an inefficient method to solve a calculation, for example using improper fractions to work out $4 \times 8\frac{3}{15}$
- Children may make errors converting between improper fractions and mixed numbers.

Key questions

- How could you partition this mixed number?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What do you need to do if you have an improper fraction in your answer?
- Could you work it out another way? Which way is most efficient?
- Have you written your answer in its simplest form?

Possible sentence stems

- I can partition $\square \frac{\square}{\square}$ into \square and $\frac{\square}{\square}$
- When I multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.
- To multiply a mixed number by an integer, I multiply the _____ by the integer and the _____ by the integer.

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Multiply a mixed number by an integer

Key learning

- Rosie is working out $1\frac{1}{5} \times 3$

I know that $1\frac{1}{5} \times 3 = 1\frac{1}{5} + 1\frac{1}{5} + 1\frac{1}{5} = 3\frac{3}{5}$

Use Rosie's method to work out the multiplications.

$1\frac{1}{7} \times 3$

$2\frac{2}{10} \times 3$

$3 \times 5\frac{3}{10}$

$2 \times 4\frac{3}{11}$

- Amir is working out $3 \times 5\frac{1}{10}$



I will partition $5\frac{1}{10}$ into 5 and $\frac{1}{10}$ and multiply each part by 3

$3 \times 5 = 15 \quad 3 \times \frac{1}{10} = \frac{3}{10} \quad 3 \times 5\frac{1}{10} = 15\frac{3}{10}$

Use Amir's method to work out the multiplications.

$3 \times 4\frac{1}{10}$

$5\frac{3}{10} \times 3$

$6 \times 2\frac{1}{7}$

$3\frac{4}{9} \times 2$

- Whitney is working out $3 \times 2\frac{2}{5}$ by partitioning the mixed number into a whole number and a fraction.

$3 \times 2 = 6$

$3 \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$

$3 \times 2\frac{2}{5} = 6 + 1\frac{1}{5} = 7\frac{1}{5}$

Use Whitney's method to work out the multiplications.

$4 \times 2\frac{2}{5}$

$3\frac{3}{8} \times 4$

$5 \times 3\frac{7}{9}$

$3 \times 5\frac{3}{7}$

- Scott uses improper fractions to work out $4 \times 1\frac{3}{8} = 5\frac{1}{2}$

$4 \times 1\frac{3}{8} = 4 \times \frac{11}{8} = \frac{44}{8} = 5\frac{4}{8} = 5\frac{1}{2}$

Use Scott's method to work out the multiplications.

$5 \times 1\frac{5}{6}$

$1\frac{9}{10} \times 3$

$6 \times 2\frac{3}{5}$

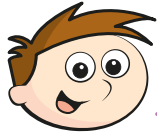
$4 \times 12\frac{7}{8}$

Is Scott's method always efficient?

Multiply a mixed number by an integer

Reasoning and problem solving

Teddy is working out $5 \times 2\frac{3}{5}$



$$5 \times 2 = 10$$

$$5 \times \frac{3}{5} = \frac{15}{5}$$

The answer is $10\frac{15}{5}$

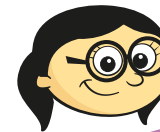
Do you agree with Teddy?

Explain your answer.

Yes, but he has not simplified his answer.

$\frac{15}{5} = 3$, so the answer is 13

Annie is working out $2 \times 3\frac{2}{7}$



If I work out $3 \times 2\frac{2}{7}$, I will get the same answer.

Do you agree with Annie?

Explain your answer.

No

$$2 \times 3\frac{2}{7} = 6\frac{4}{7}$$

$$3 \times 2\frac{2}{7} = 6\frac{6}{7}$$

Jack runs $2\frac{2}{3}$ miles three times per week.

Mo runs $3\frac{3}{4}$ miles twice a week.

Who runs further during the week?

Explain your answer.

Jack

Find the missing numbers.

$$2\frac{\square}{8} \times \square = 7\frac{7}{8}$$

Explain how you worked out the missing numbers.

$$2\frac{5}{8} \times 3 = 7\frac{7}{8}$$

Calculate a fraction of a quantity

Notes and guidance

In this small step, children calculate a fraction of a quantity, building on understanding from previous years. The step focuses on using concrete and pictorial representations to support learning.

Children begin by using real-life objects or counters and sharing them into equal groups. This helps children to identify the relationship between dividing by the denominator and multiplying by the numerator. They start by finding unit fractions of amounts and, when they are secure in their understanding, move on to non-unit fractions.

Children will build on this understanding in the next step, in which they focus on more abstract methods.

Things to look out for

- Children may divide by the numerator rather than by the denominator.
- Children may find it more difficult to find non-unit fractions of amounts, as it involves more than one step and requires more cognitive load.
- If using place value counters, children may not exchange and may believe they cannot find, for example, $\frac{1}{4}$ of 52

Key questions

- How can you share the counters equally?
- How do you know the counters are in equal groups?
- If you know $\frac{1}{\square}$ of a number, how do you find $\frac{2}{\square}$ of the number?
- What do you need to do when you cannot share your tens counters equally?
- How do you find a fraction of an amount?

Possible sentence stems

- If I know $\frac{1}{\square}$ of a quantity, then to find $\frac{\square}{\square}$ I need to multiply by _____
- To find $\frac{3}{4}$ of _____, I need to divide by _____ and multiply by _____
- I need to divide by the _____ and multiply by the _____

National Curriculum links

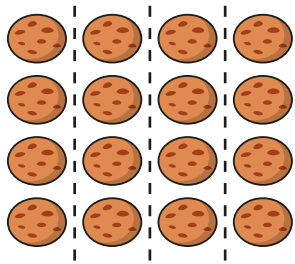
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Calculate a fraction of a quantity

Key learning

- Dora is sharing 16 cookies between 4 friends.

She needs to find $\frac{1}{4}$ of 16



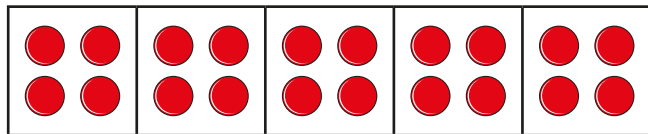
I put my 16 cookies into 4 equal groups.
 $\frac{1}{4}$ of 16 = 4

Use Dora's method to work out the fractions of amounts.

| | | | |
|---------------------|---------------------|---------------------|----------------------|
| $\frac{1}{5}$ of 20 | $\frac{1}{5}$ of 30 | $\frac{1}{6}$ of 30 | $\frac{1}{10}$ of 30 |
|---------------------|---------------------|---------------------|----------------------|

- The bar model shows 20 counters shared equally into 5 groups.

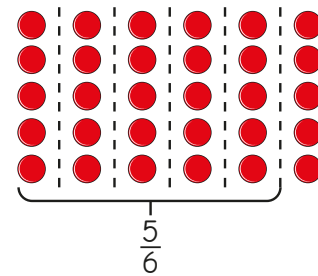
Use the bar model to find the fractions of amounts.



- ▶ $\frac{1}{5}$ of 20 ▶ $\frac{2}{5}$ of 20 ▶ $\frac{3}{5}$ of 20 ▶ $\frac{4}{5}$ of 20 ▶ $\frac{5}{5}$ of 20

What do you notice?

- Tommy uses an array of counters to find $\frac{5}{6}$ of 30

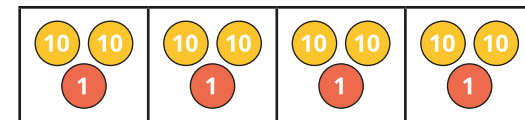


I split the counters into 6 equal groups.
 $\frac{5}{6}$ of 30 = 25

Use Tommy's method to work out the fractions of amounts.

| | | | |
|---------------------|---------------------|---------------------|----------------------|
| $\frac{2}{5}$ of 30 | $\frac{4}{5}$ of 30 | $\frac{2}{3}$ of 30 | $\frac{7}{10}$ of 30 |
|---------------------|---------------------|---------------------|----------------------|

- Nijah uses place value counters to find $\frac{1}{4}$ of 84 = 21



Use Nijah's method to work out the fractions of amounts.

| | | | |
|---------------------|---------------------|---------------------|---------------------|
| $\frac{1}{4}$ of 48 | $\frac{3}{4}$ of 88 | $\frac{2}{3}$ of 96 | $\frac{4}{5}$ of 55 |
| $\frac{3}{4}$ of 92 | $\frac{5}{6}$ of 72 | $\frac{4}{7}$ of 84 | $\frac{3}{4}$ of 76 |

Why were the last calculations more challenging?

Calculate a fraction of a quantity

Reasoning and problem solving

Alex is finding $\frac{3}{5}$ of 15



$$15 \div 3 = 5 \quad \frac{3}{5} \text{ of } 15 = 5$$

Do you agree with Alex?

Explain your answer.



No

Ron is trying to find $\frac{4}{8}$ of 20



It is impossible to find $\frac{4}{8}$ of 20, as I cannot split 20 into 8 equal groups.

Do you agree with Ron?

Explain your answer.



No

Sort the calculations into the table.



$$\frac{1}{2} \text{ of } 16$$

$$\frac{1}{5} \text{ of } 30$$

$$\frac{3}{7} \text{ of } 14$$

$$\frac{2}{9} \text{ of } 36$$

| Equal to 6 | Equal to 8 |
|------------|------------|
| | |

Write another calculation in each column.

How many different answers can you find?



$$= 6: \frac{1}{5} \text{ of } 30, \frac{3}{7} \text{ of } 14$$

$$= 8: \frac{1}{2} \text{ of } 16, \frac{2}{9} \text{ of } 36$$

multiple possible answers, e.g.

$$= 6: \frac{1}{2} \text{ of } 12, \frac{2}{10} \text{ of } 30$$

$$= 8: \frac{1}{4} \text{ of } 32, \frac{4}{5} \text{ of } 10$$

Fraction of an amount

Notes and guidance

In this small step, children find fractions of amounts using more pictorial and abstract methods, rather than relying on concrete resources.

Bar models are useful tools to help represent this mathematical concept and can also help to show links between finding unit fractions of amounts and non-unit fractions of amounts. Children initially use times-table facts, then move on to solve calculations that go beyond these. Once children are secure in finding non-unit fractions of amounts, they compare two calculations, for example $\frac{2}{3}$ of 30 and $\frac{4}{5}$ of 20

The learning from this step is built upon in Step 6, when children find the whole from a fractional part.

Things to look out for

- Children may divide by the numerator and not by the denominator.
- Children may find it more difficult to find non-unit fractions of amounts, as this involves more than one step and greater cognitive load.
- Children may need support to find fractions of amounts that go beyond known times-table facts.

Key questions

- How can you represent this in a bar model?
- What is the relationship between $\frac{1}{\square}$ of a number and $\frac{2}{\square}$ of a number?
- What is the first step to solve this calculation?
What is the next step to solve this calculation?
- How do you find a fraction of an amount?
- How can you find a fraction of a 3-digit number?

Possible sentence stems

- To find $\frac{\square}{\square}$ of _____, I need to divide by _____ and multiply by _____
- To find a fraction of an amount, I need to divide by the _____ and multiply the result by the _____

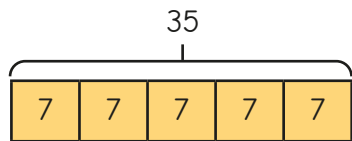
National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Fraction of an amount

Key learning

- Esther is finding $\frac{1}{5}$ of 35



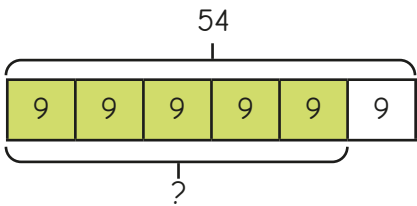
$$35 \div 5 = 7$$

$$\frac{1}{5} \text{ of } 35 = 7$$

Use Esther's method to work out the fractions of amounts.

| | | | |
|---------------------|---------------------|----------------------|---------------------|
| $\frac{1}{5}$ of 45 | $\frac{1}{7}$ of 35 | $\frac{1}{10}$ of 80 | $\frac{1}{9}$ of 81 |
|---------------------|---------------------|----------------------|---------------------|

- Mo is finding $\frac{5}{6}$ of 54



$$54 \div 6 = 9$$

$$9 \times 5 = 45$$

$$\frac{5}{6} \text{ of } 54 = 45$$

- ▶ Complete the sentence.
To find a fraction of an amount, I need to divide by the _____ and multiply by the _____
- ▶ Use Mo's method to work out the fractions of amounts.

| | | | |
|---------------------|---------------------|---------------------|---------------------|
| $\frac{2}{3}$ of 21 | $\frac{4}{5}$ of 40 | $\frac{3}{5}$ of 60 | $\frac{5}{7}$ of 42 |
|---------------------|---------------------|---------------------|---------------------|

- Write $<$, $>$ or $=$ to compare the fractions of amounts.

$$\frac{1}{3} \text{ of } 33 \quad \bigcirc \quad \frac{2}{5} \text{ of } 25$$

$$\frac{2}{7} \text{ of } 140 \quad \bigcirc \quad \frac{3}{4} \text{ of } 76$$

- Work out the fractions of amounts.

▶ $\frac{1}{3}$ of 30 $\frac{2}{3}$ of 30 $\frac{3}{3}$ of 30

▶ $\frac{1}{5}$ of 35 $\frac{2}{5}$ of 35 $\frac{3}{5}$ of 35 $\frac{4}{5}$ of 35 $\frac{5}{5}$ of 35

What patterns do you notice?

- Brett is finding $\frac{2}{3}$ of 618

$$618 \div 3 = 206$$

$$206 \times 2 = 412$$

$$\frac{2}{3} \text{ of } 618 = 412$$

Use Brett's method to work out the fractions of amounts.

| | | | |
|----------------------|----------------------|----------------------|------------------------|
| $\frac{2}{3}$ of 924 | $\frac{5}{6}$ of 126 | $\frac{3}{5}$ of 205 | $\frac{7}{9}$ of 6,498 |
|----------------------|----------------------|----------------------|------------------------|

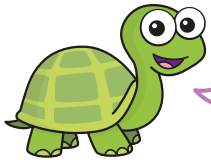
- Draw bar models to help work out the fractions of quantities.

| | | |
|------------------------|-------------------------|----------------------------|
| $\frac{3}{7}$ of 21 kg | $\frac{4}{5}$ of 100 cm | $\frac{5}{12}$ of 1,440 ml |
|------------------------|-------------------------|----------------------------|

Fraction of an amount

Reasoning and problem solving

Tiny is working out $\frac{6}{6}$ of 36



I will divide by the denominator and then multiply by the numerator.

No

Do you think that this is the most efficient method?

Explain your answer.

Find three possible ways to make the statement true.

$$\frac{3}{\square} \text{ of } \underline{\hspace{2cm}} = 15$$

Compare answers with a partner.

What do you notice?

multiple possible answers, e.g.

$$\frac{3}{4} \text{ of } 20 = 15$$

There are 32 boys and girls in a class.

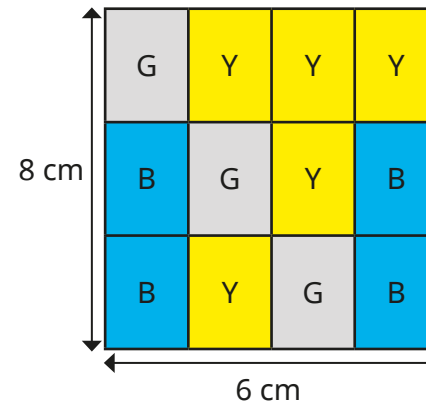
$\frac{7}{16}$ of the class are boys.

How many more girls than boys are there?

How did you work it out?

4

Find the area of each colour (grey, blue and yellow).



grey: 12 cm^2
blue: 16 cm^2
yellow: 20 cm^2

Find the whole

Notes and guidance

In this small step, children build on their understanding of finding a fraction of an amount, as they use a fraction of an amount to find the whole.

Children start with finding the whole from a unit fraction, initially using counters and bar models for support. They identify that if they know one equal part, they can use multiplication to find the whole. Once this is secure, children move on to finding the whole from a non-unit fraction. They should start by identifying what one part is to help them work out the whole.

Things to look out for

- Children may misinterpret the question by trying to find the fraction of the number given, instead of using the number to find the whole.
- Children may mix up finding one part with finding the whole.
- When dealing with a non-unit fraction, children may divide by the denominator to find one part, rather than dividing by the numerator.

Key questions

- What is the same and what is different about finding a fraction of an amount and finding the whole?
- If you know that one equal part is _____, what must all the other parts be?
- If you know one equal part, how can you work out the whole?
- If you know what _____ equal parts are, how can you find what one part is?
- Is your answer going to be greater or less than _____? How do you know?

Possible sentence stems

- If _____ is one equal part, all the parts must be _____
- If $\frac{1}{\square}$ is _____, then the whole is _____ \times _____ = _____
- If _____ is _____ parts, then one part is _____

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Find the whole

Key learning

- The counters in the bar model show that $\frac{1}{4}$ of a quantity is 5

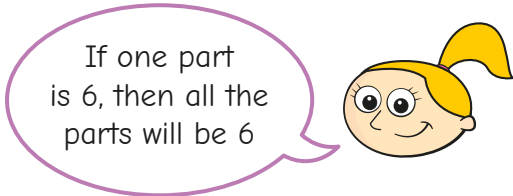
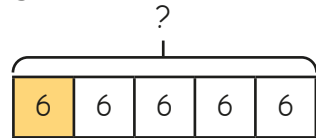


Use the bar model to work out the fractions of the same quantity.

▶ $\frac{2}{4} = \underline{\hspace{2cm}}$ ▶ $\frac{3}{4} = \underline{\hspace{2cm}}$ ▶ $\frac{4}{4}$ or 1 whole = $\underline{\hspace{2cm}}$

- Eva uses a bar model to help work out the missing amount.

$\frac{1}{5}$ of $\underline{\hspace{2cm}} = 6$



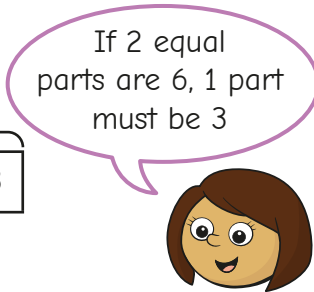
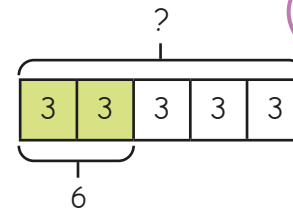
$6 \times 5 = 30$ $\frac{1}{5}$ of $30 = 6$

Use Eva's method to work out the missing amounts.

▶ $\frac{1}{5}$ of $\underline{\hspace{2cm}} = 9$ ▶ $\frac{1}{7}$ of $\underline{\hspace{2cm}} = 10$
 ▶ $\frac{1}{8}$ of $\underline{\hspace{2cm}} = 3$ ▶ $\frac{1}{12}$ of $\underline{\hspace{2cm}} = 2$

- Kim uses a bar model to help work out the missing amount.

$\frac{2}{5}$ of $\underline{\hspace{2cm}} = 6$

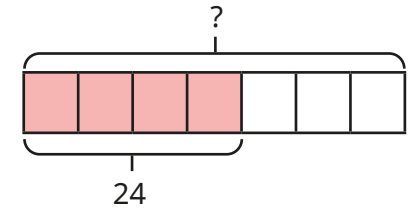
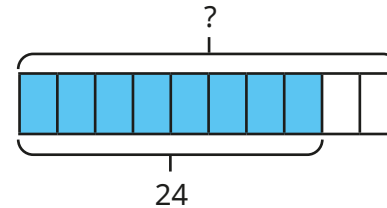


$6 \div 2 = 3$
 $3 \times 5 = 15$
 $\frac{2}{5}$ of $15 = 6$

Use Kim's method to work out the missing amounts.

▶ $\frac{2}{5}$ of $\underline{\hspace{2cm}} = 8$ ▶ $\frac{3}{7}$ of $\underline{\hspace{2cm}} = 18$
 ▶ $\frac{4}{5}$ of $\underline{\hspace{2cm}} = 20$ ▶ $\frac{6}{7}$ of $\underline{\hspace{2cm}} = 54$

- Use the bar models to find the wholes.



- Jack has a bottle of juice.
 There is $\frac{3}{5}$ of a bottle left.
 There is 150 ml of juice left in the bottle.
 How much juice was in the bottle when it was full?

Find the whole

Reasoning and problem solving

Amir and Tiny are working out the missing amount.

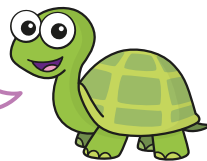
$$\frac{7}{5} \text{ of } \underline{\hspace{2cm}} = 42$$



I think the answer is 30

Amir

The answer cannot be 30. It must be greater than 42



Tiny

Who do you agree with?
Explain your answer.

Amir

Rosie takes a bottle of water to school with her.

She drinks $\frac{1}{3}$ of the water in the morning.

She drinks $\frac{1}{4}$ of the bottle at lunchtime.

So far, she has drunk 210 ml of water.

How much water was in her bottle when it was full?

360 ml

If $\frac{1}{8}$ of A = 12, find the values of A, B and C.

$$\frac{5}{8} \text{ of } A = \frac{3}{4} \text{ of } B = \frac{1}{6} \text{ of } C$$

A = 96

B = 80

C = 360

Use fractions as operators

Notes and guidance

In this small step, children revisit and compare their learning from earlier in the block as they look at fractions as operators. They should recognise the connection between finding a fraction of an amount and multiplying a fraction by an integer.

Firstly, children are encouraged to both find fractions of amounts and multiply fractions, and to identify patterns. It may be appropriate to recap converting improper fractions to whole numbers/mixed numbers. Children should also recognise that commutativity of multiplication can be used, for example $\frac{1}{3}$ of 6 is the same as $6 \times \frac{1}{3}$. They also explore when it would be more efficient to choose each method, using their knowledge of factors.

Things to look out for

- Children may need support to recognise the link between “of” and \times .
- Children may make errors if their times-tables knowledge is insecure.
- Children may choose the less appropriate method and face difficult calculations as a result.

Key questions

- What is the same about _____ of _____ and _____ \times _____?
- Is the denominator of the fraction a factor of the number you are multiplying by? Why is this important?
- Which is the most efficient method? How do you know?
- How would you write this improper fraction as a whole number/mixed number?
- When is it more efficient to multiply fractions?
- When is it more efficient to find a fraction of an amount?

Possible sentence stems

- $\frac{\square}{\square} \times \text{_____}$ is the same as $\frac{\square}{\square}$ of _____
- _____ is a factor of _____, so I can divide _____ by _____

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number (Y4)

Use fractions as operators

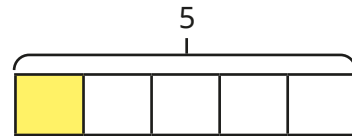
Key learning

- Use the bar models to work out the calculations.

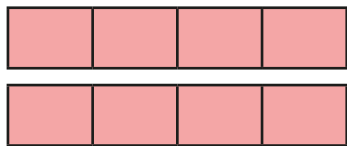
▶ 5 lots of $\frac{1}{5}$



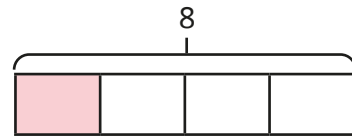
$\frac{1}{5}$ of 5



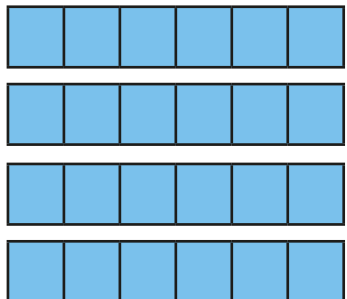
▶ 8 lots of $\frac{1}{4}$



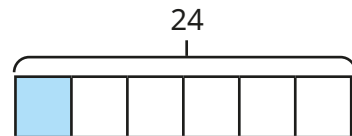
$\frac{1}{4}$ of 8



▶ $\frac{1}{6} \times 24$



$\frac{1}{6}$ of 24



What do you notice?

- Use bar models to work out the calculations.

▶ $\frac{2}{3}$ of 15

▶ $\frac{4}{5}$ of 30

▶ $\frac{3}{10}$ of 20

$\frac{2}{3} \times 15$

$30 \times \frac{4}{5}$

$\frac{3}{10} \times 20$

Which bar model did you find easiest to draw? Was this the same for each question?

- Match the calculations that give the same answer.

$\frac{4}{5}$ of 45

$10 \times \frac{2}{10}$

$\frac{2}{10}$ of 10

$\frac{1}{9} \times 81$

$\frac{7}{8}$ of 56

$\frac{4}{5} \times 45$

$\frac{1}{9}$ of 81

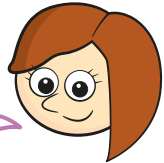
$56 \times \frac{7}{8}$

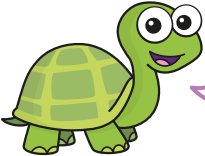
Work out the answer to each calculation.
In each pair, which was the easier calculation to work out?

Use fractions as operators


Reasoning and problem solving

Tiny is working out $\frac{4}{5}$ of 15
 Rosie is working out $15 \times \frac{4}{5}$


 We will get the same answer.
 Rosie

 No, we won't. We are working out different calculations.
 Tiny


Who do you agree with?
 Explain your answer.



Rosie

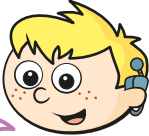
Is the statement true or false? 

$\frac{9}{10}$ of 20 = $20 \times \frac{4}{10} + 20 \times \frac{5}{10}$

Explain your answer. 

True

Max is thinking of a 2-digit number between 20 and 30
 He finds $\frac{2}{3}$ of the number.

 My new number has a digit sum of 7

What number did Max start with?

24

Spring Block 3

Decimals and percentages

Small steps

Step 1

Decimals up to 2 decimal places

Step 2

Equivalent fractions and decimals (tenths)

Step 3

Equivalent fractions and decimals (hundredths)

Step 4

Equivalent fractions and decimals

Step 5

Thousandths as fractions

Step 6

Thousandths as decimals

Step 7

Thousandths on a place value chart

Step 8

Order and compare decimals (same number of decimal places)

Small steps

Step 9 Order and compare any decimals with up to 3 decimal places

Step 10 Round to the nearest whole number

Step 11 Round to 1 decimal place

Step 12 Understand percentages

Step 13 Percentages as fractions

Step 14 Percentages as decimals

Step 15 Equivalent fractions, decimals and percentages

Decimals up to 2 decimal places

Notes and guidance

In Year 4, children represented tenths and hundredths as decimals and fractions. By the end of this small step, children will be more familiar with numbers with up to 2 decimal places, with thousandths being introduced later in the block.

Using a hundred piece of base 10 as 1 whole, a ten piece as a tenth and a one piece as a hundredth shows children that they can exchange, for example, 10 tenths for 1 whole, or 10 hundredths for 1 tenth. A hundred square where each part represents 1 hundredth, or 0.01, can also help children to see the relationship between a hundredth, a tenth and a whole.

Children make decimal numbers using place value counters in a place value chart and read and write the numbers, as well as working out the value of each digit in the number. They also explore partitioning decimal numbers in a variety of ways.

Things to look out for

- When reading or writing a number, children may say “one point thirty-five” instead of “one point three five”.
- When there are hundredths but no tenths in a number, children may forget to include the zero placeholder in the tenths column.

Key questions

- How can you represent this number using a place value chart?
- What is the same and what is different about a tenth and a hundredth?
- What is the value of the digit _____ in the number _____?
- Can you partition the decimal number _____ in different ways?
- How many tens are there in 100?
How many ones are there in 10/100?
- How many 0.1s are there there are in 1?
How many 0.01s are there in 0.1/1?

Possible sentence stems

- _____ tenths/hundredths are equivalent to _____ wholes/tenths.
- The value of the digit _____ in the number _____ is _____

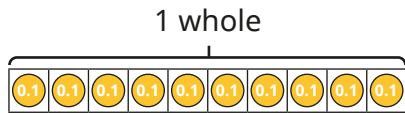
National Curriculum links

- Read, write, order and compare numbers with up to 3 decimal places

Decimals up to 2 decimal places

Key learning

- Whitney shares 1 whole into 10 equal parts.

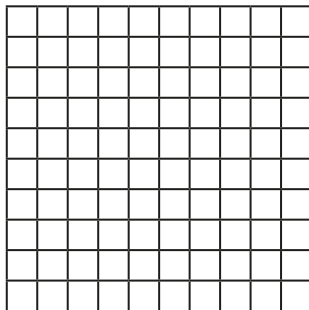


Use the bar model to complete the sentences.

- ▶ One part is worth _____ tenth, which is written as _____
- ▶ Seven parts are worth _____ tenths, which is written as _____

- Jack uses a hundred square to represent 1 whole.

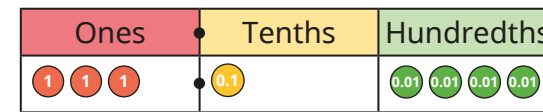
Each part represents 0.01



Use the hundred square to complete the sentences.

- ▶ One part is worth _____ hundredth, which is written as _____
- ▶ Five parts are worth _____ hundredths, which is written as _____
- ▶ The whole square is worth _____ hundredths, which is written as _____

- Huan uses place value counters to make the number 3.14



Use place value counters to make the numbers.



- Complete the sentence to describe the underlined digit in each number.



The value of the digit _____ in the number _____ is _____

- Fill in the missing numbers.

- ▶ $0.83 = \underline{\quad} + 0.03 = \underline{\quad}$ tenths and 3 hundredths
- ▶ $0.83 = 0.7 + \underline{\quad} = 7$ tenths and _____ hundredths

How many other ways can you partition 0.83?

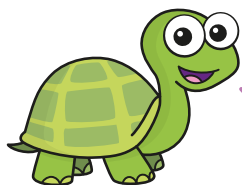
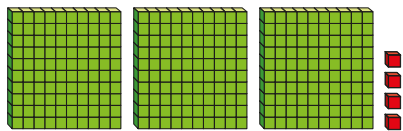
Decimals up to 2 decimal places

Reasoning and problem solving

Filip is using base 10 to make decimal numbers.

He uses a hundred piece to represent 1, a ten piece to represent 0.1 and a one piece to represent 0.01

He makes this number.



Filip has made the number 3.4

Do you agree with Tiny?

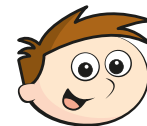
Explain your answer.



No



Match the numbers to the children.



Teddy

My number has the same amount of tens and tenths.



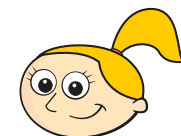
Amir

My number has one decimal place.



Rosie

My number has 2 hundredths.



Eva

My number has 6 tenths.

46.2

2.64

46.02

40.46

Teddy: 40.46

Amir: 46.2

Rosie: 46.02

Eva: 2.64

Equivalent fractions and decimals (tenths)

Notes and guidance

In Year 4, children learnt about tenths as fractions as well as decimals. In this small step, children consolidate their understanding of equivalent fractions and decimals when working with tenths.

Children start by exploring equivalent fractions and decimals within 1, before extending this to numbers greater than 1. Place value counters, bead strings, straws and number lines are all good representations for tenths. Remind children that when 1 is split into 10 equal parts, then one of those parts is called a tenth, which could also be written as 0.1, making $\frac{1}{10}$ and 0.1 equivalent.

It is important children practise counting up in 0.1s and crossing 1 whole, making sure they do not say “zero point nine, zero point ten, zero point eleven ...”. For numbers greater than 1, for example 1.2, children should see this written as 1.2, $1\frac{2}{10}$ and $\frac{12}{10}$.

Things to look out for

- Children may count up in 0.1s to 0.10 (“zero point ten”).
- Children may confuse the words “tens” and “tenths”.
- With numbers greater than 1, children may find mixed numbers easier than improper fractions, or vice versa.

Key questions

- What is the same/different about fractions and decimals?
- If a whole is split into 10 equal parts, what is each part worth?
- What does “equivalent” mean?
- What decimal is equivalent to the fraction _____?
- What fraction is equivalent to _____ 0.1s?
- When counting up in $\frac{1}{10}$ s/0.1s, what happens after $\frac{9}{10}$ /0.9?
- How many tenths are there in the number _____?

Possible sentence stems

- The fraction _____ is equivalent to the decimal _____
- The decimal _____ is equivalent to the fraction _____
- There are ten _____ in 1 whole.

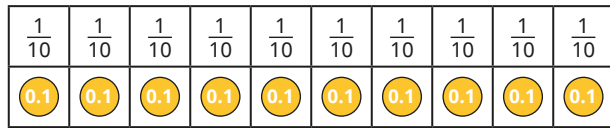
National Curriculum links

- Read and write decimal numbers as fractions

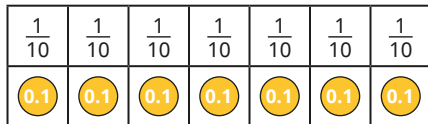
Equivalent fractions and decimals (tenths)

Key learning

- Kim uses a bar model to show the equivalence of 0.1 and $\frac{1}{10}$



She then uses a bar model to make a number.



Complete the sentences to describe Kim's number.

- ▶ The fraction represented is _____
 - ▶ The decimal represented is _____
 - ▶ The fraction _____ is equivalent to the decimal _____
- Ron uses a bead string to represent 1 whole.



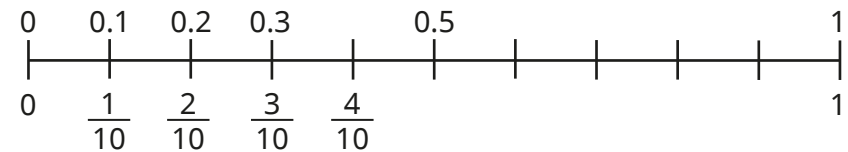
Then he uses the bead string to represent another number.



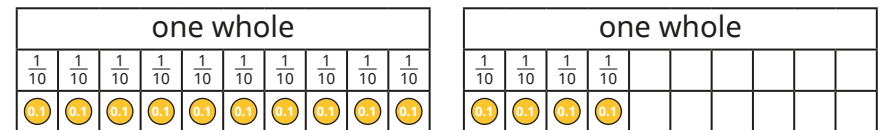
Write the number that Ron has represented.

Give your answer as a fraction and as a decimal.

- Complete the number line.

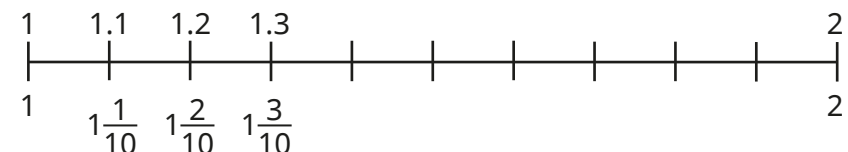


- The bar models show that $1\frac{4}{10}$ is equal to 1.4



Draw your own bar models to help complete the statements.

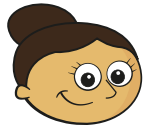
- ▶ $1\frac{3}{10} =$ _____
 - ▶ $2.6 =$ _____
 - ▶ $\frac{32}{10} =$ _____
- Complete the number line.



Equivalent fractions and decimals (tenths)

Reasoning and problem solving

Dora, Annie and Dexter are describing the same number.



I think it is $\frac{17}{10}$

Dora



I think it is 1.7

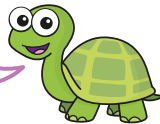
Annie



I think it is 1 and $\frac{7}{10}$

Dexter

They have all said different numbers, so someone must be wrong.

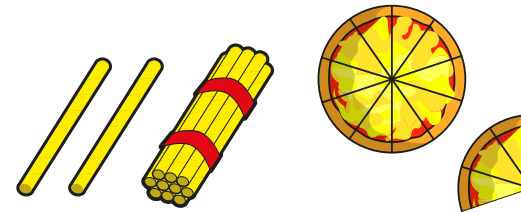
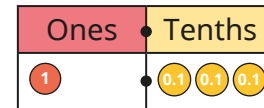
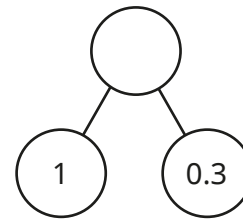


Do you agree with Tiny?

Explain your answer.

No

Which representation is the odd one out?



Explain your answer.

The straws are the odd one out.

They represent 1.2 or $1\frac{2}{10}$

The other models all represent 1.3 or $1\frac{3}{10}$

Equivalent fractions and decimals (hundredths)

Notes and guidance

In this small step, children extend the learning of the previous step to explore equivalent fractions and decimals when looking at hundredths.

Using a hundred square with a value of 1, and each part worth $\frac{1}{100}$ or 0.01, helps children's understanding of hundredths in relation to the whole. They also see that because $\frac{10}{100}$ is equivalent to $\frac{1}{10}$, decimal numbers with 2 decimal places can be partitioned into tenths and hundredths, for example $\frac{32}{100} = \frac{3}{10} + \frac{2}{100}$ and $0.32 = 0.3 + 0.02$. Learning then extends to decimals and fractions greater than 1. Children see fractions greater than 1 whole as both mixed numbers and improper fractions, for example $1.03 = 1\frac{3}{100} = \frac{103}{100}$

Things to look out for

- Children may confuse the words “hundreds” and “hundredths”.
- When converting a decimal into tenths and hundredths, children may confuse the two, for example $0.23 = \frac{2}{100} + \frac{3}{10}$
- When counting up in 0.01s or $\frac{1}{100}$ s, at 1 whole, children may incorrectly say, for example, 0.23 as “zero point twenty-three”.

Key questions

- What is the same/different about fractions/decimals?
- What fraction is the decimal _____ equivalent to?
- What decimal is the fraction _____ equivalent to?
- What is the value of the digit _____ in _____?
- What fractions can the decimal _____ be partitioned into?
- How many tenths are equal to 1 whole?
- How many hundredths are equal to 1 whole?
- How many hundredths are equal to 1 tenth?

Possible sentence stems

- The fraction/decimal _____ is equivalent to the decimal/fraction _____
- There are _____ tenths and _____ hundredths in _____
- _____ hundredths is equivalent to _____ tenths.

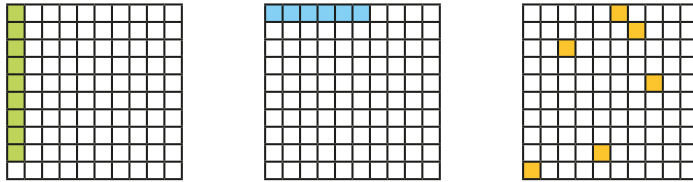
National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- Read and write decimal numbers as fractions

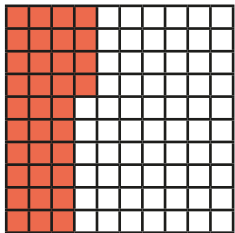
Equivalent fractions and decimals (hundredths)

Key learning

- Each square in the hundred grid represents 1 hundredth. What fraction and what decimal of each hundred square is shaded?



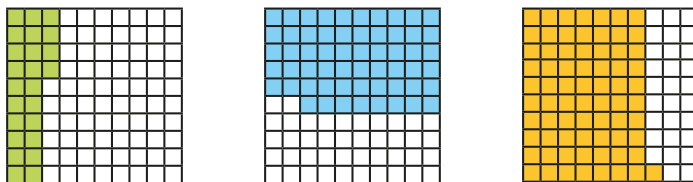
- Esther knows that each column in the hundred square is worth $\frac{1}{10}$. She shades some squares and describes the number.



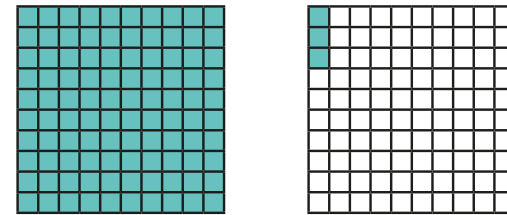
There are $\frac{3}{10}$ and $\frac{4}{100}$ shaded.
This shows the decimals $0.3 + 0.04$

$$\frac{34}{100} = 0.34$$

Write the equivalent fractions and decimals shown by each hundred square.



- Nijah shades two hundred squares to make a number greater than 1



Write Nijah's number as a fraction and as a decimal.
Shade hundred squares to show each number.

$$2\frac{3}{100}$$

$$1.08$$

$$1.3$$

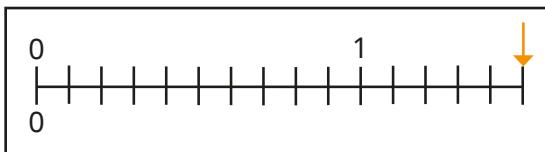
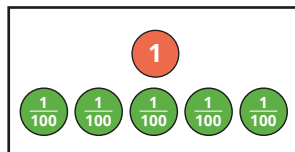
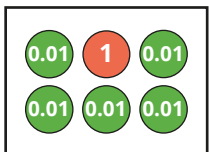
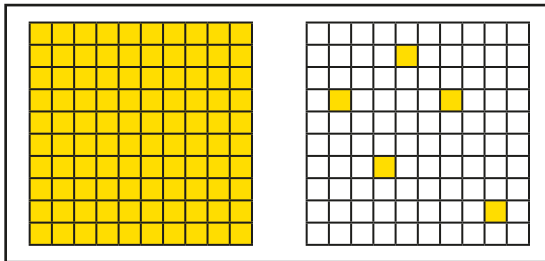
$$1\frac{89}{100}$$

- Shade hundred squares to show 1.4 and 1.04
Discuss with a partner what is the same and what is different about the two numbers.
- Write $\frac{117}{100}$ as a mixed number and as a decimal number.

Equivalent fractions and decimals (hundredths)

Reasoning and problem solving

Which representation is the odd one out?



Explain your answer.

The number line is the odd one out.

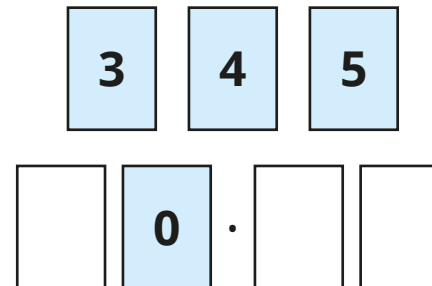
Is the statement true or false?

$$2\frac{9}{100} > 2.3$$

Explain your answer.

False

Use the digit cards to complete the decimal number.



List all the possible numbers you can make.

Write each decimal as a mixed number.

30.45, 30.54, 40.35,
40.53, 50.34, 50.43

$30\frac{45}{100}$, $30\frac{54}{100}$,

$40\frac{35}{100}$, $40\frac{53}{100}$,

$50\frac{34}{100}$, $50\frac{43}{100}$

Equivalent fractions and decimals

Notes and guidance

In this small step, children look at equivalent fractions and decimals, specifically focusing on halves, quarters, fifths and tenths. They relate this to earlier learning from Key Stage 2, when they divided 100 into 2, 4, 5 and 10 equal parts. By seeing 1 whole divided into 2, 4, 5 and 10 equal parts on a number line, children will see the value of these fractions.

They also apply their understanding of equivalent fractions/decimals from previous learning to this step. Once confident with unit fraction equivalents, children can then explore non-unit fractions such as $\frac{3}{4}$ and $\frac{2}{5}$. Fraction walls can be used to remind children of equivalent fractions such as $\frac{4}{10} = \frac{2}{5}$, which will help with their understanding.

Things to look out for

- Children may not count the intervals on a number line correctly and confuse the number of divisions with the number of intervals.
- Children may misinterpret numerators and denominators, for example writing $\frac{1}{5}$ as 1.5 or $\frac{3}{4}$ as 3.4

Key questions

- What is 1 whole shared equally into 2/4/5/10 equal parts?
- How can you tell what each interval on the number line is worth?
- What decimal is equivalent to the fraction _____?
- What fraction is the decimal _____ equivalent to?
- What is the same and what is different about the fraction _____ and the decimal _____?

Possible sentence stems

- The decimal _____ is equivalent to the fraction _____
- _____ hundredths is equivalent to _____
- If I know that _____ is equivalent to _____, then I also know that _____ is equivalent to _____

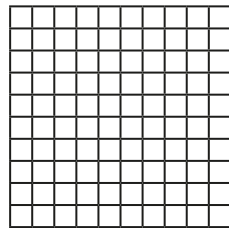
National Curriculum links

- Read and write decimal numbers as fractions
- Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

Equivalent fractions and decimals

Key learning

- Shade $\frac{1}{2}$ of the hundred square.



Use the hundred square to complete the equivalent fraction.

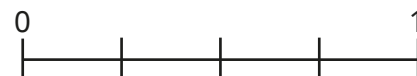
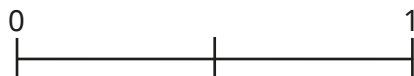
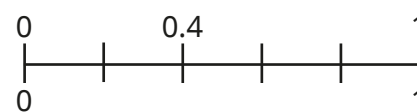
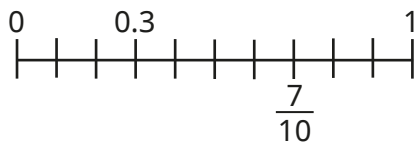
$$\frac{1}{2} = \frac{\square}{100}$$

Write the fraction as a decimal.

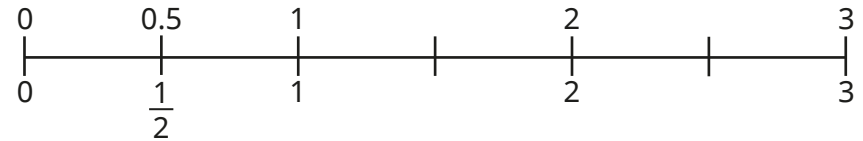
- Shade hundred squares to represent the fractions and write the equivalent fractions and decimals.

▶ $\frac{1}{10}$ ▶ $\frac{1}{4}$ ▶ $\frac{1}{5}$

- Label the missing decimals and fractions on the number lines.

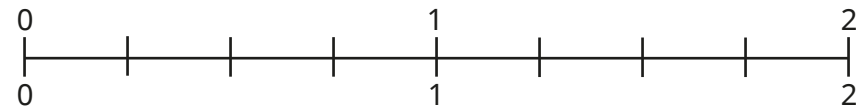


- Ron has started counting in halves on a number line.



Complete Ron's number line.

- Fill in the missing fractions and decimals on the number line.



- What decimals and fractions are the arrows pointing to?



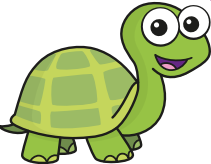
- Work out the equivalent fraction or decimal for each number.

Give fraction answers as both mixed numbers and improper fractions.

▶ $\frac{2}{5}$ ▶ 1.1 ▶ $\frac{13}{10}$ ▶ $1\frac{3}{4}$ ▶ 2.5

Equivalent fractions and decimals

Reasoning and problem solving



$\frac{1}{10}$ is equivalent to 0.1, so $\frac{1}{4}$ is equivalent to 0.4

Do you agree with Tiny?
Explain your answer.

No

Is the statement true or false?

2.5 as a fraction is $\frac{2}{5}$

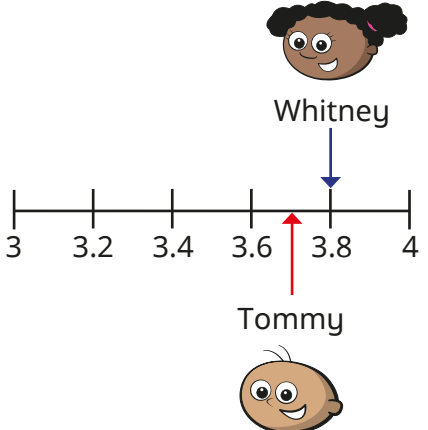
Explain your answer.

False

Tommy and Whitney are working on the same number line.

Tommy draws an arrow halfway between 3.6 and 3.8

Whitney draws an arrow to 3.8



What decimal is halfway between Tommy and Whitney's arrows?
Write the decimal as a mixed number.

3.75

$3\frac{3}{4}$

Thousandths as fractions

Notes and guidance

In this small step, children encounter the idea of thousandths for the first time.

Begin by reminding children that a tenth is 1 whole split into 10 equal parts, a hundredth is 1 whole split into 100 equal parts, and therefore a thousandth is 1 whole split into 1,000 equal parts. Different representations can be used to model this idea, such as a thousand piece of base 10 representing the whole and a one piece representing a thousandth.

Once children are familiar with the idea of a thousandth, they use place value counters to represent them. Exchanging counters helps children to see that there are 10 thousandths in a hundredth, meaning 9 thousandths is smaller than 1 hundredth. Finally, they partition thousandths into tenths, hundredths and thousandths, for example $\frac{342}{1000} = \frac{3}{10} + \frac{4}{100} + \frac{2}{1000}$

Things to look out for

- Children may confuse the words “thousand” and “thousandth”.
- As 1,000 is greater than 100, children may think that $\frac{1}{1000}$ is greater than $\frac{1}{100}$

Key questions

- What is a thousandth?
- How are thousandths similar to/different from tenths/hundredths?
- How many thousandths are there in 1 whole?
- How many thousandths are there in 1 hundredth?
- How many thousandths are there in 1 tenth?
- How can you partition _____ thousandths?
- What fraction is made up of _____ tenths, _____ hundredths and _____ thousandths?
- Which is greater, 1 hundredth or 9 thousandths? How do you know?

Possible sentence stems

- There are _____ thousandths in _____
- $\frac{\square}{1000}$ is equivalent to $\frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1000}$

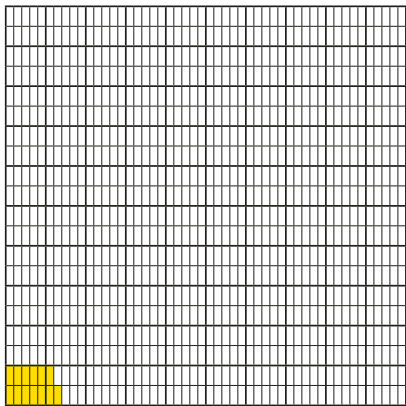
National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

Thousandths as fractions

Key learning

- Here is a thousand square.



13 parts are shaded.
This represents $\frac{13}{1000}$

What fractions are represented by these amounts?

22 shaded parts

150 shaded parts

1 shaded part

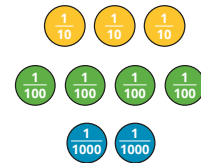
- Use the fact that $\frac{1}{10} = \frac{10}{100}$ and $\frac{1}{100} = \frac{10}{1000}$ to complete the equivalent fractions.

▶ $\frac{1}{10} = \frac{\square}{1000}$

▶ $\frac{4}{100} = \frac{\square}{1000}$

▶ $\frac{800}{1000} = \frac{\square}{100} = \frac{\square}{10}$

- Scott uses place value counters to partition $\frac{342}{1000}$

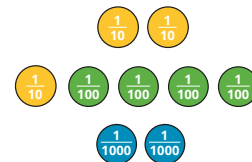


$$\frac{342}{1000} = \frac{3}{10} + \frac{4}{100} + \frac{2}{1000}$$

Use Scott's method to partition the fractions.

▶ $\frac{267}{1000}$ ▶ $\frac{607}{1000}$ ▶ $\frac{53}{1000}$

- Sam uses place value counters to partition $\frac{342}{1000}$ flexibly.



$$\frac{342}{1000} = \frac{2}{10} + \frac{14}{100} + \frac{2}{1000}$$

Use Sam's method to partition the fractions flexibly.

▶ $\frac{267}{1000}$ ▶ $\frac{607}{1000}$ ▶ $\frac{53}{1000}$

- Write <, > or = to complete the statements.

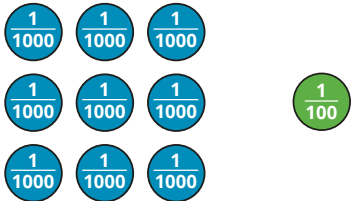
$\frac{9}{1000} \bigcirc \frac{8}{100}$

$\frac{1}{10} \bigcirc \frac{2}{100}$

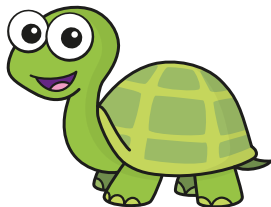
$\frac{1}{10} \bigcirc \frac{100}{1000}$

Thousandths as fractions

Reasoning and problem solving



$\frac{9}{1000}$ is greater than $\frac{1}{100}$ because 9 is greater than 1 and 1,000 is greater than 100

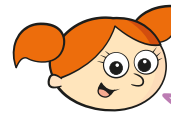


Do you agree with Tiny?
Explain your answer.

No

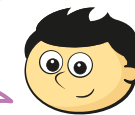
$\frac{1}{100}$ is equivalent to $\frac{10}{1000}$, which is greater than $\frac{9}{1000}$

Alex, Jack and Whitney are partitioning $\frac{367}{1000}$



Alex

It can be partitioned into 3 tenths, 6 hundredths and 7 thousandths.



Jack

It can be partitioned into 2 tenths, 16 hundredths and 7 thousandths.



Whitney

It can be partitioned into 3 tenths and 67 thousandths.

All three children are correct.

Who do you agree with?
Explain your answer.

Thousandths as decimals

Notes and guidance

In this small step, children continue to explore the idea of thousandths, by representing them in decimal form.

Children learn that $0.001 = \frac{1}{1000}$ is a tenth the size of $0.01 = \frac{1}{100}$.

Exchanging place value decimal counters from 1 down to 0.001 helps them to understand the relationship between the different decimals. They use number lines labelled in hundredths and see that by splitting each section into 10 equal parts, the number line now shows thousandths.

Children flexibly partition decimal numbers with 3 decimal places. Using place value counters and exchanging between the values will help them to understand this concept.

Things to look out for

- Children may confuse the words “thousand” and “thousandth”.
- Children may use the incorrect number of placeholders, leading to the incorrect number being written.
- Children may think that, for example, $0.01 + 0.004 = 0.0005$ because they just add the non-zero digits.

Key questions

- What does each digit in a decimal number represent?
- How are 0.001s similar to $\frac{1}{1000}$ s? How are they different?
- How many 0.001s are there in 1 whole?
- How many 0.001s are there in 0.01?
- How many 0.001s are there in 0.1?
- How can you represent 0.001s on a number line?

Possible sentence stems

- _____ is 10 times greater than _____
- _____ is one-tenth the size of _____
- There are _____ _____ in _____

National Curriculum links

- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- Read, write, order and compare numbers with up to 3 decimal places

Thousandths as decimals

Key learning

- The diagram shows the relationship between tenths, hundredths and thousandths.



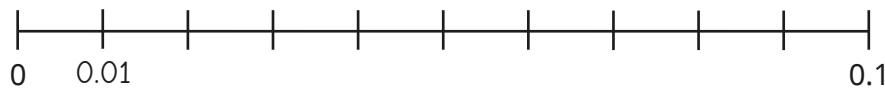
Complete the sentences in as many ways as possible.

_____ is one-tenth the size of _____

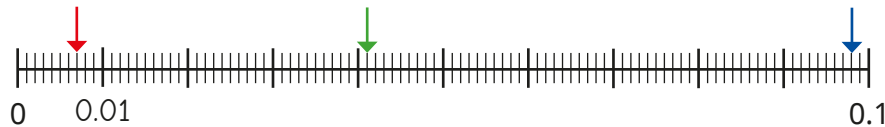
_____ is 10 times the size of _____

- Rosie is counting up from 0 to 0.1 in hundredths on a number line.

Finish labelling her number line.



She then splits each section into 10 equal parts.



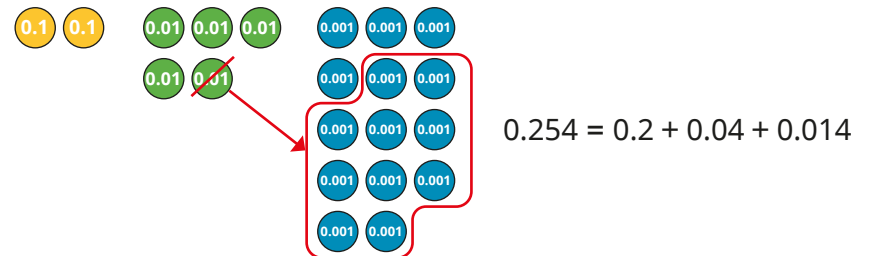
The first arrow is pointing to 0.007

What numbers are the other arrows pointing to?

- The number 0.254 is made up of 2 tenths, 5 hundredths and 4 thousandths.

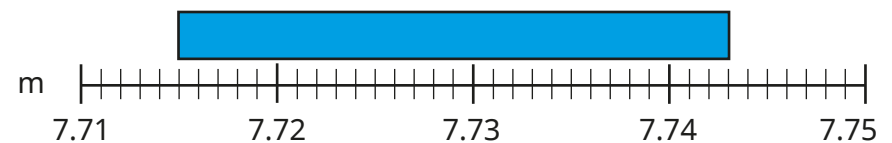


The same number can also be made like this, by exchanging 1 hundredth for 10 thousandths.



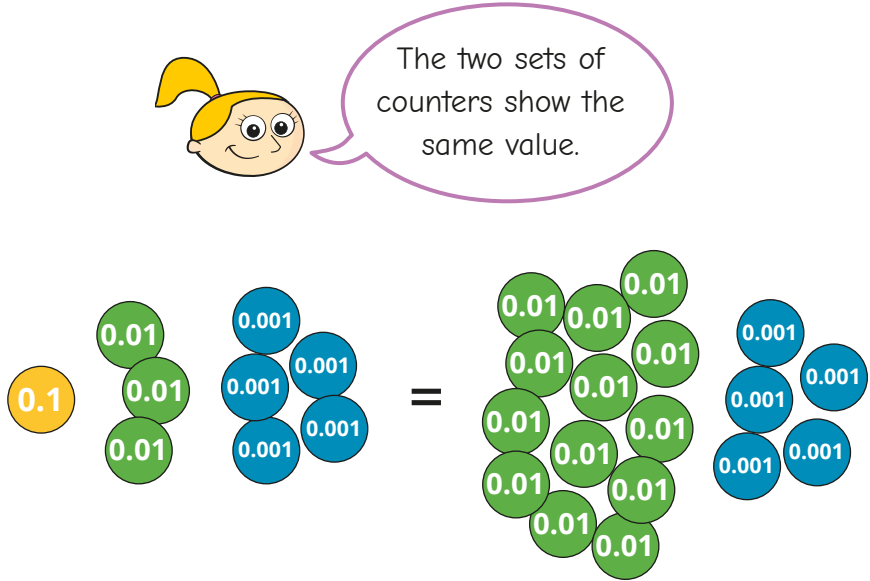
Partition the number 0.428 in three different ways.

- How long is the rectangle?



Thousandths as decimals

Reasoning and problem solving




The two sets of counters show the same value.


Do you agree with Eva?
Explain your answer.
Write this value as a decimal and as a fraction.

Yes | 0.135, $\frac{135}{1000}$


Three children are partitioning the number 0.504



Jo

$$0.504 = 0.5 + 0.004$$


Amir

$$0.504 = 0.3 + 0.2 + 0.004$$


Teddy

$$0.504 = \frac{5}{10} + \frac{4}{1000}$$

Who is correct?
Explain your answer.

They are all correct. Jo has partitioned the number as decimals. Amir has partitioned the number as decimals in a different way. Teddy has partitioned the number as fractions.

Thousandths on a place value chart

Notes and guidance

In this small step, children continue to explore the idea of thousandths, by representing numbers with up to 3 decimal places on a place value chart. This is the first time this column of the chart will have been shown to the children and some recap work on the place value chart may be needed.

Show children decimal numbers represented on the place value chart with place value counters and ask what decimal number has been made. Then provide children with numbers for them to make using place value counters. They should see that a decimal such as 0.012 is shown on a place value chart as one 0.01 counter in the tenths column and two 0.001 counters in the thousandths column.

Children partition decimal numbers in a variety of ways. Making the number first with place value counters and then exchanging for different values will help them flexibly partition decimals.

Things to look out for

- Children may be unsure how to use placeholders if there is an empty column, for example 5 tenths and 7 thousandths = 0.507
- Children may see, for example, $\frac{23}{1000}$ and start by putting 2 in the thousandths column and then 3 in the ten-thousandths column (0.0023).

Key questions

- What is a thousandth?
- How many thousandths are equivalent to 1 hundredth?
- How can you represent this decimal number on a place value chart?
- What is the value of the digit _____ in _____?
How does a place value chart help you?
- What do you need to do when there are no counters in a column?

Possible sentence stems

- _____ ones, _____ tenths, _____ hundredths and _____ thousandths make the decimal number _____
- _____ can be partitioned into _____ + _____ + _____
- I know that _____ is equivalent to _____ because ...

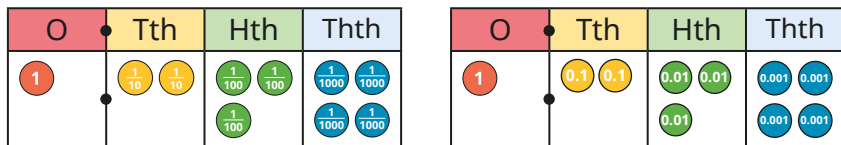
National Curriculum links

- Read, write, order and compare numbers with up to 3 decimal places
- Solve problems involving numbers up to 3 decimal places

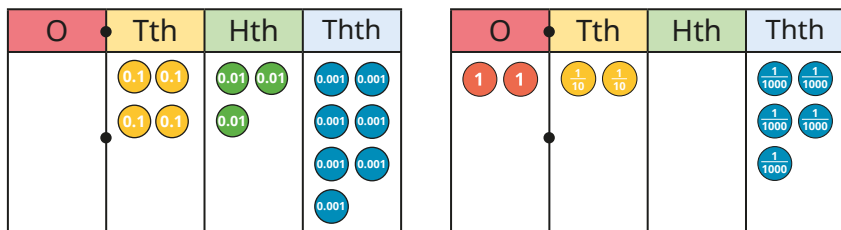
Thousandths on a place value chart

Key learning

- What is the same and what is different about these place value charts?



- Complete the sentences to describe each number.



There are _____ ones.

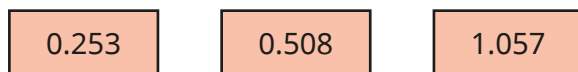
There are _____ tenths.

There are _____ hundredths.

There are _____ thousandths.

The number represented is _____

- Make each number on a place value chart.



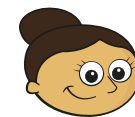
- $\frac{12}{1000}$ can be partitioned into $\frac{1}{100}$ and $\frac{2}{1000}$

Partition these numbers into hundredths and thousandths.

Use a place value chart to help you.



- Dora and Ron have partitioned 0.132 in different ways.



Dora

$$0.132 = 0.1 + 0.03 + 0.002$$

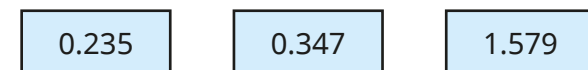


Ron

$$0.132 = 0.1 + 0.02 + 0.012$$

Use a place value chart and counters to show that both children are correct.

- Use a place value chart to help you partition the numbers in different ways.



Compare answers with a partner.

Thousandths on a place value chart

Reasoning and problem solving

Brett has eight plain counters.



He makes numbers using the place value chart.

| O | Tth | Hth | Thth |
|---|-----|-----|------|
| | | | |

6.11

0.116

At least three columns contain counters.

What is the greatest number he can make?

What is the smallest number he can make?

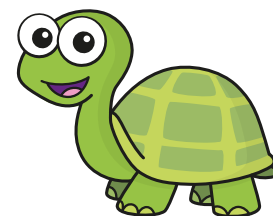
Tiny puts the fraction $\frac{45}{1000}$ into a place value chart.



| O | Tth | Hth | Thth |
|---|-----|-----|------|
| 0 | 0 | 0 | 4 |

5

There are 3 zeros in 1,000, so I need 3 zeros at the start of my number.



Do you agree with Tiny?

Explain your answer.



No

Order and compare decimals (same number of decimal places)

Notes and guidance

In Year 4, children ordered and compared decimal numbers with up to 2 decimal places. In this small step, that learning is extended to include numbers with 3 decimal places. For this step, the number of decimal places in each number will be the same.

Representations such as place value charts and counters and number lines can be used to support children's understanding.

To begin with, the numbers will have different digits in the column with the greatest value. Children identify the column with the greatest value in each number and identify which number has the greater digit in this column. They then order numbers in a similar way. They progress to two numbers with the same digit in the column with the greatest value so that they use the next column (or the next) to determine which number has the greater value.

Things to look out for

- Children may not appreciate that they must start with the column with the greatest value, leading to misconceptions such as thinking 0.299 is greater than 0.312
- Children may have forgotten the terms “ascending” and “descending”.

Key questions

- How do you compare two numbers?
- Which column in the place value chart do you need to look at first?
- How can you compare two numbers that have the same number of tenths/hundredths?
- Which number is greater, _____ or _____?
- What does “ascending”/“descending” mean?

Possible sentence stems

- I need to start by looking at the column with the _____ place value.
- To compare _____ and _____, I need to first look at the _____ column.
- If the digits in the _____ column are the same, I need to look at the _____ column.

National Curriculum links

- Read, write, order and compare numbers with up to 3 decimal places
- Solve problems involving numbers up to 3 decimal places

Order and compare decimals (same number of decimal places)

Key learning

- Which is the greater number, 0.6 or 0.4?

How do you know?

Which is the greater number, 0.14 or 0.17?

How do you know?

- Make the numbers 0.452 and 0.364 on a place value chart.

How do your place value charts show that 0.452 is greater than 0.364?

Talk about it with a partner.

- Write $>$ or $<$ to compare the numbers.

Use a place value chart and counters to help you.

$$0.465 \bigcirc 0.913 \quad 0.067 \bigcirc 0.029$$

$$1.546 \bigcirc 0.894 \quad 0.071 \bigcirc 0.007$$

- Write the numbers in ascending order.

0.379 0.209 0.693 0.895 0.172

- Use place value charts to make the numbers 0.569 and 0.571

How do your place value charts show that 0.569 is less than 0.571?

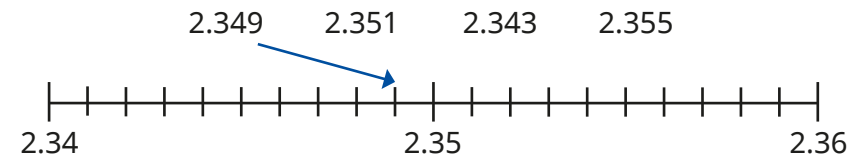
- Write $>$ or $<$ to compare the numbers.

Use a place value chart to help you.

$$0.756 \bigcirc 0.719 \quad 0.658 \bigcirc 0.659$$

$$0.021 \bigcirc 0.022 \quad 1.036 \bigcirc 1.035$$

- Eva is using a number line to order some numbers.



Draw arrows to show the positions of the other numbers.

Then write the numbers in ascending order.

- Write the numbers in ascending order.

65.394 65.309 63.999 65.493

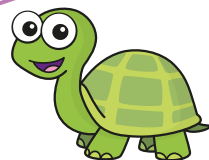
Order and compare decimals (same number of decimal places)

Reasoning and problem solving

Esther uses counters and a place value chart to make two numbers.

| O | Tth | Hth | Thth |
|---|-----|---------|---------|
| | ● ● | ● | ● |
| | ● | ● ● ● ● | ● ● ● ● |

The second number has 8 counters in it and the first only has 4, so the second number is greater.



Do you agree with Tiny?
Explain your answer.



No

Whitney, Mo and Tommy are each thinking of a number.



Whitney

My number is 3.465

My number is 3.455



Mo

3.456, 3.457, 3.458,
3.459, 3.46, 3.461,
3.462, 3.463, 3.464



Tommy

My number is between Whitney and Mo's numbers.

What number could Tommy be thinking of?

Order and compare any decimals with up to 3 decimal places

Notes and guidance

In this small step, children compare decimal numbers that have a different number of decimal places.

A common misconception with this learning is thinking that numbers with more decimal places are greater, for example $0.365 > 0.41$.

Using place value counters on a place value chart to build numbers supports children in developing their understanding. They should recognise that 0.41 has more tenths than 0.365 – it does not matter that it has fewer decimal places.

Using place value charts supports children to recognise that they need to start comparing the numbers from the place value column that has the highest value, and that if this is the same, they need to look at the next column.

When progressing to ordering sets of numbers, encourage children to work systematically through the list, starting by comparing the place value column that has the greatest value, then working their way down.

Things to look out for

- Children may read 1.234 as “one point two hundred and thirty-four” and therefore assume it is greater than 1.3
- When ordering decimals, children may not write all of the numbers from the question in their answer.

Key questions

- What is the same and what is different about 1.4 and 1.305?
- What are the digits in each number worth?
- How can you represent these numbers on a place value chart?
- Which place value column in the chart has the greatest value? Which has the next greatest value?
- How can a place value chart help to show you which number is greater?
- How can you work systematically to order numbers in a list?

Possible sentence stems

- _____ is greater/smaller than _____ because ...
- The decimal _____ has a greater value than the decimal _____
- _____ tenths/hundredths/thousandths are greater than _____ tenths/hundredths/thousandths, so _____ is greater than _____

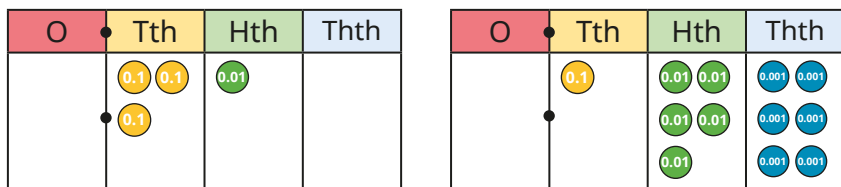
National Curriculum links

- Read, write, order and compare numbers with up to 3 decimal places
- Solve problems involving numbers up to 3 decimal places

Order and compare any decimals with up to 3 decimal places

Key learning

- Rosie has made the numbers 0.31 and 0.156 on place value charts.



Which number is greater? How do you know?

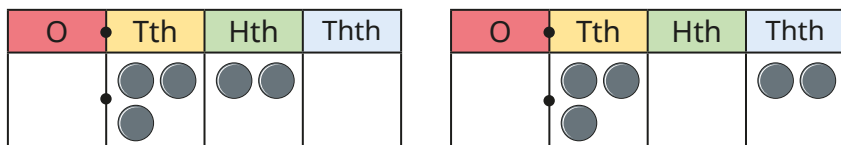
- Write $>$ or $<$ to compare the numbers.

Use a place value chart and counters to help you.

$$0.65 \bigcirc 0.7 \qquad 1.5 \bigcirc 0.988$$

$$0.406 \bigcirc 0.32 \qquad 0.9 \bigcirc 0.769$$

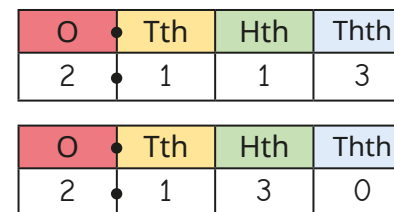
- The place value charts show the numbers 0.32 and 0.302



What is the same and what is different about the numbers?

Which number is greater? How do you know?

- Max has written the numbers 2.113 and 2.13 in place value charts.



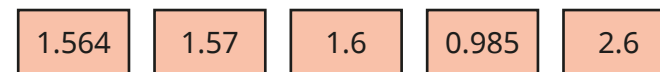
Which of the numbers is greater? How do you know?

Which place value column did you need to compare?

- Write $>$ or $<$ to compare the numbers.

$$2.4 \bigcirc 2.38 \qquad 1.865 \bigcirc 1.87 \qquad 3.079 \bigcirc 3.7$$

- Write the numbers in ascending order.

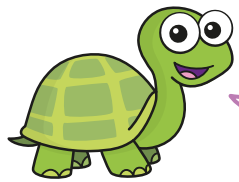


- Put these lengths in order, from longest to shortest.



Order and compare any decimals with up to 3 decimal places

Reasoning and problem solving



5.35 is greater than 5.4 because 35 is greater than 4

Do you agree with Tiny?

Explain why.



No

Amir is thinking of two numbers.

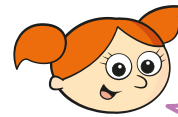
Use the clues to work out what his numbers could be.

- The greater number has 2 decimal places.
- The smaller number has 3 decimal places.
- You need to look at the hundredths column to compare them.

How many answers can you find?



multiple possible answers, e.g. 0.23 and 0.219



I have put some numbers in ascending order.

3.015

$3\frac{51}{1000}$



3.105

$3\frac{51}{100}$

Alex has missed one number out.

What could the number be?

What could the number **not** be?

multiple possible answers, e.g. 3.052, 3.053, 3.054, 3.104

less than or equal to 3.051 or greater than or equal to 3.105

Round to the nearest whole number

Notes and guidance

Earlier in Year 5, children rounded whole numbers within 1,000,000. In Year 4, they rounded decimal numbers to the nearest whole number. In this small step, children round numbers with 1 and 2 decimal places to the nearest whole number. This extends to rounding to 1 decimal place in the next step.

Begin by recapping what whole numbers are and which integers are either side of a decimal number. Place value charts and counters allow children to explore how far away each integer is on either side of the decimal number. Using a number line supports understanding of rounding and helps determine which whole number is closer. Children decide whether the number is greater or smaller than the halfway point between the integers. When the number is exactly halfway between two whole numbers, explain that the convention is to round to the greater of the two, for example 6.5 rounds to 7

Things to look out for

- Children may see 6.15 as “six point fifteen” and round to 7 because 15 is greater than 5
- Children may not think of zero as a whole number.
- The words “round down” can result in children rounding incorrectly, for example rounding 7.2 to 6 rather than 7

Key questions

- Which integers (whole numbers) lie either side of this decimal number?
- Where would the decimal _____ go on this number line?
- How can you work out which whole number a decimal number is closer to?
- Which whole number is the decimal _____ closer to? How do you know?
- What is halfway between these two whole numbers?
- When a decimal number has fewer than 5 tenths, does it round to the next or previous whole number? How do you know?

Possible sentence stems

- The whole numbers either side of _____ are _____ and _____
- _____ is closer to _____ than _____
- _____ rounded to the nearest whole number is _____

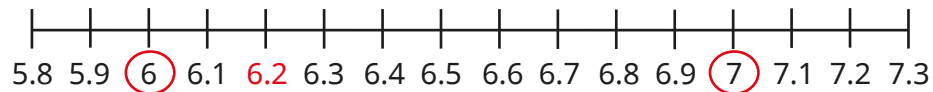
National Curriculum links

- Round decimals with 2 decimal places to the nearest whole number and to 1 decimal place

Round to the nearest whole number

Key learning

- Huan has used a number line to find that the whole numbers either side of 6.2 are 6 and 7



Use a number line to find the whole numbers that are either side of each decimal number.

| | | | | | |
|-----|------|-----|------|------|------|
| 4.8 | 12.4 | 9.9 | 2.21 | 6.78 | 0.74 |
|-----|------|-----|------|------|------|

- Jack makes the number 3.8 using place value counters.

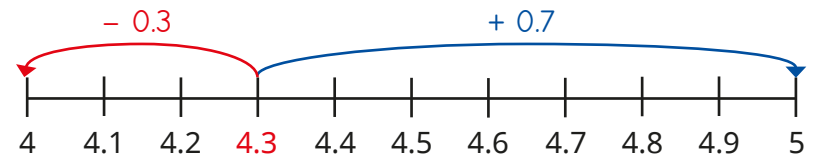


I need to add two 0.1s to make 4, but I need to subtract eight 0.1s to make 3. So 3.8 is closer to 4

Use Jack's method to decide what integer each number is closest to.

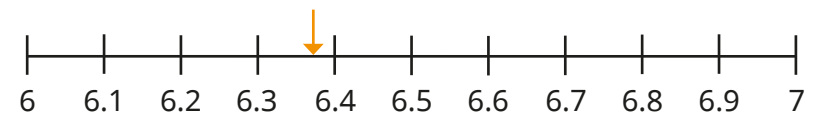
| | | | | |
|-----|-----|-----|------|------|
| 3.3 | 5.2 | 6.9 | 14.2 | 28.7 |
|-----|-----|-----|------|------|

- Dani is rounding 4.3 to the nearest whole number using a number line.



4.3 rounded to the nearest whole number is 4

- Use the number line to round 4.9, 4.1 and 4.6 to the nearest whole number.
- Which integer does 4.5 round to? Why?
- The number line shows that 6.37 is less than 6.5, so rounds to 6 to the nearest whole number.




Use a number line to round the numbers to the nearest whole number.

- ▶ 6.71 ▶ 3.81 ▶ 5.59 ▶ 10.05 ▶ 7.49

Round to the nearest whole number

Reasoning and problem solving

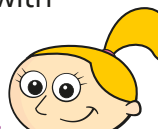
Amir is thinking of a number with 2 decimal places.



Rounded to the nearest whole number, my number is 5

Amir

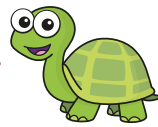
Eva is thinking of a number with 1 decimal place.




Rounded to the nearest whole number, my number is 6

Eva

Eva's number must be greater than Amir's number.

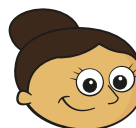


Do you agree with Tiny?
Explain your answer.




Yes

Dora is thinking of a number with 1 decimal place.



Rounded to the nearest whole number, my number is 10

What is the difference between the greatest and smallest possible numbers Dora could be thinking of?




0.9 (10.4 – 9.5)

Scott is thinking of a number with 2 decimal places.

When he rounds the number to the nearest whole number, the answer is zero.

What is the greatest number Scott could be thinking of?



0.49

Round to 1 decimal place

Notes and guidance

In this small step, children build on the previous step by rounding to 1 decimal place.

They see which numbers with 1 decimal place are either side of a number with 2 decimal places. From here, they work out which number with 1 decimal place is closer. As with rounding to the nearest whole number, a number line is a useful visual aid. When rounding to 1 decimal place, if the digit in the hundredths column is 5, children learn that the number rounds to the greater of the two numbers with 1 decimal place. It is important that children understand that integers, including zero, can also be written as numbers with 1 decimal place, for example $3 = 3.0$

For this step, only numbers with up to 2 decimal places will be rounded, as rounding numbers with 3 decimal places is covered in Year 6

Things to look out for

- Children may not think of zero as a whole number.
- Children may round to the whole number rather than 1 decimal place.
- The phrase “round down” can lead children to round too low, for example rounding 6.91 down to 6.8 rather than 6.9

Key questions

- How can you work out what numbers with 1 decimal place are either side of a number with two decimal places?
- Which number with 1 decimal place is your number closer to? How do you know?
- What number is halfway between the two numbers to 1 decimal place?
- How do you round a number that is halfway between the two numbers to 1 decimal place?

Possible sentence stems

- The numbers with 1 decimal place either side of _____ are _____ and _____
_____ is closer to _____ than _____
_____ rounded to 1 decimal place is _____
- Halfway between _____ and _____ is _____

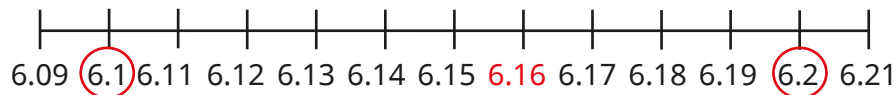
National Curriculum links

- Round decimals with 2 decimal places to the nearest whole number and to 1 decimal place

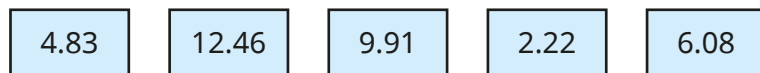
Round to 1 decimal place

Key learning

- Aisha has used a number line to find which numbers with 1 decimal place lie either side of 6.16



Use a number line to find the numbers with 1 decimal place that lie either side of each number.



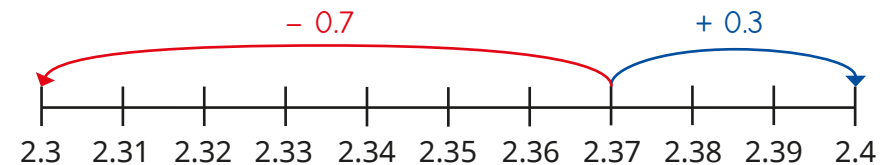
- Here is the number 3.43



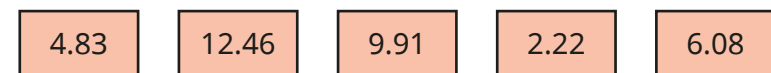
- ▶ How can you use the place value counters to show that 3.43 rounds to 3.4 to 1 decimal place?
- ▶ Use place value counters to round the numbers to 1 decimal place.



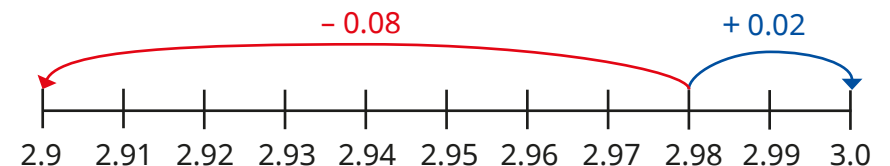
- Teddy has used a number line to find that 2.37 rounded to 1 decimal place is 2.4



Use Teddy's method to round the numbers to 1 decimal place.



- How does the number line show that 2.98 rounds to 3.0 to 1 decimal place?

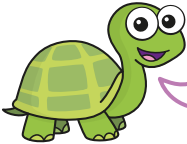


Round the numbers to 1 decimal place.




Round to 1 decimal place

Reasoning and problem solving




Rounding to 1 decimal place is the same as rounding to the nearest tenth.

Do you agree with Tiny?
Explain your answer.




Yes

Mo is thinking of a number.




Rounded to the nearest whole number, my number is 4
Rounded to the nearest tenth, my number is 3.8

Write at least four different numbers that Mo could be thinking of.




multiple possible answers, e.g.
3.75, 3.79, 3.81, 3.84
Some children may include answers such as 3.845


Whitney is thinking of a number between 11 and 20



My number has 2 decimal places.
When I round it to 1 decimal place, I get the same answer as when I round it to the nearest whole number.



What could Whitney's number be?
Is there more than one possible answer?
Talk about it with a partner.



multiple possible answers, e.g.
14.95, 17.97, 19.04

Understand percentages

Notes and guidance

In this small step, children are introduced to percentages for the first time.

Children learn that “per cent” relates to “number of parts per 100”. If the whole is split into 100 equal parts, then each part is worth 1%. Hundred squares and 100-piece bead strings or Rekenreks are useful representations for exploring this concept. This idea can also be linked to previous learning by comparing to hundredths being 1 part out of a whole that is split into 100 equal parts; this will be covered in greater detail in the following steps.

Using bar models, the learning extends to 1 whole being split into 10 equal parts, allowing children to explore multiples of 10%. Children then estimate 5% on a bar model split into 10 equal parts by splitting a section in half, for example 35% is three full sections and half of the next section.

Things to look out for

- Children may think that 1% means 1 part, regardless of whether there are 100 parts in total or not.
- Children may forget to write the % symbol.
- When seeing 1 part out of a whole that has been split into 10 parts, children may believe this is 1% rather than 10%.

Key questions

- How many parts is the square split into?
- How many parts per hundred are shaded/not shaded?
- What percentage of the square is shaded/not shaded?
- What does “100%” mean?
- How many parts is the bar model split into?
- If the whole bar represents 100%, what is each part worth?

Possible sentence stems

- If the whole is shared into 100 equal parts, then each part represents _____%.
- If the whole is shared into 10 equal parts, then each part represents _____%.
- _____ out of _____ equal parts are shaded.
The percentage shaded is _____%.

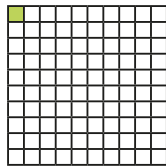
National Curriculum links

- Recognise the per cent symbol (%) and understand that per cent relates to “number of parts per 100”, and write percentages as a fraction with denominator 100, and as a decimal fraction

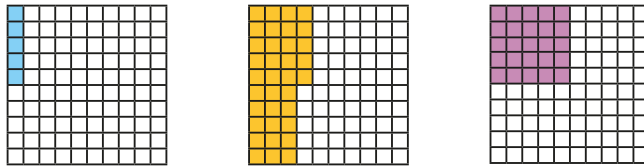
Understand percentages

Key learning

- The hundred square has 1 part shaded. This is 1%.



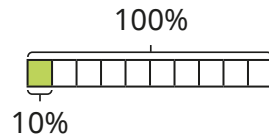
How many parts of each hundred square are shaded?



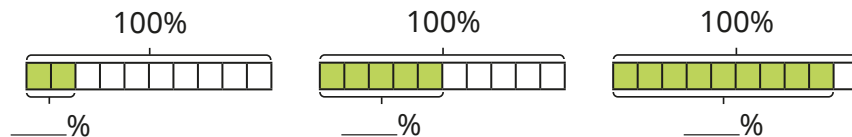
What percentage of each hundred square is shaded?

- The bar model has been split into 10 equal parts and 1 part is shaded.

This is 10%:

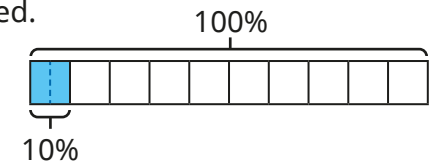


What percentage of each bar model is shaded?



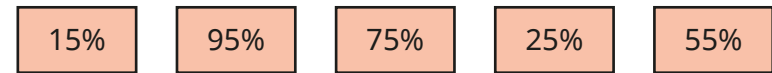
- Esther's bar model has 10% shaded.

She draws a line to split the shaded part into two equal parts.



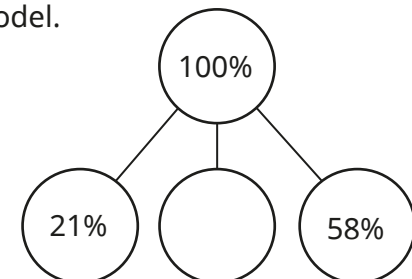
What is each of the smaller parts worth?

- Draw bar models to show the percentages.



- There are 100 children in a school. All the children have either a school dinner or a packed lunch. 47 children have a packed lunch. What percentage of children in the school have a school dinner?

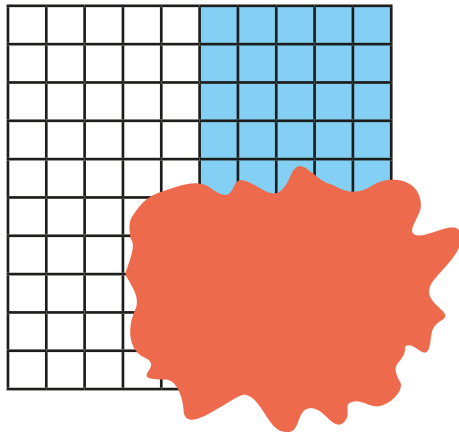
- Complete the part-whole model.



Understand percentages

Reasoning and problem solving

Filip has spilt paint on his hundred square.



Complete the sentences to describe what percentage is shaded.

It could be _____%.

It must be _____%.

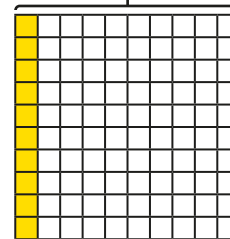
It cannot be _____%.

multiple possible answers, e.g.
It could be 25%.
It must be less than 55%.
It cannot be 100%.

Whitney and Brett have drawn diagrams showing percentages.

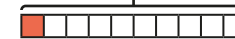
Whitney

100%



Brett

100%



I have shaded more parts than Brett, so my percentage is greater.

Do you agree with Whitney?

Explain your answer.

No

Percentages as fractions

Notes and guidance

In this small step, children continue to explore percentages by comparing them to fractions.

In the previous step, children saw that a percentage was a number of parts per hundred. This links to seeing a percentage as a fraction with a denominator of 100. This learning extends to 10% being equivalent to $\frac{1}{10}$ and therefore 20% equivalent to $\frac{2}{10}$ and so on. Children use a fraction wall to split 100% into different-sized groups and so work out the percentage equivalents of fractions, for example $\frac{1}{4}$ is 100% split into 4 groups, $100 \div 4 = 25$, so $\frac{1}{4} = 25\%$.

The focus of this step is percentages and fractions within 1 whole only. Decimal equivalents will be introduced in the next step.

Things to look out for

- Children may think that the numerator of any fraction is the same as the percentage, for example $\frac{9}{10} = 9\%$.
- Not knowing common equivalent fractions to those with a denominator of 100 will make finding those percentages hard, for example not knowing $\frac{1}{4} = \frac{25}{100}$ will make finding $\frac{1}{4} = 25\%$ difficult.

Key questions

- What is a percentage?
- If the whole is split into 100 equal parts, then what percentage is _____ parts equivalent to?
- How are percentages and fractions similar? How are they different?
- What is 100 divided by 2/4/5/10?
- What is _____ as a percentage?
- What is one half of 100? What is $\frac{1}{2}$ as a percentage?

Possible sentence stems

- _____% is equivalent to $\frac{\square}{100}$
- The fraction _____ is equivalent to _____%.

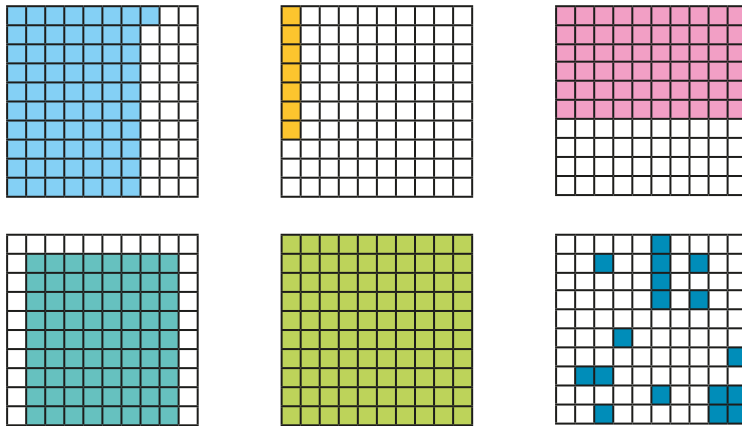
National Curriculum links

- Recognise the per cent symbol (%) and understand that per cent relates to “number of parts per 100”, and write percentages as a fraction with denominator 100, and as a decimal fraction
- Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

Percentages as fractions

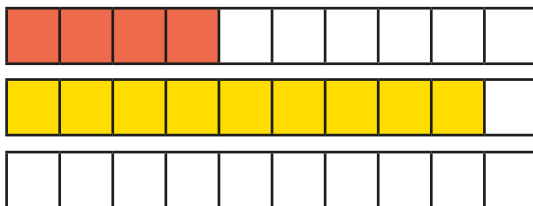
Key learning

- Complete the sentence to find what fraction and what percentage of each hundred square has been shaded.

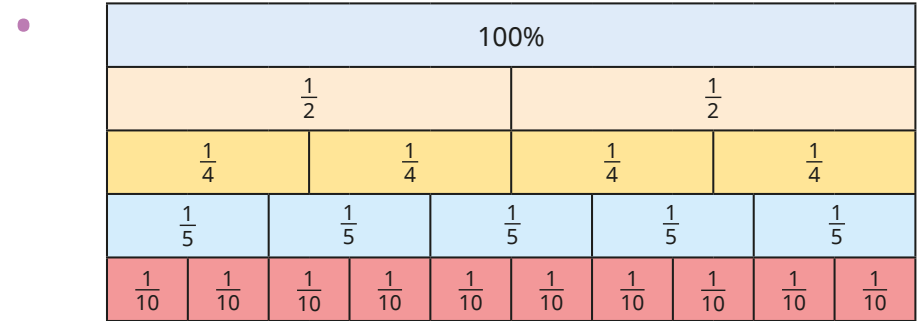


_____ parts out of 100 = $\frac{\square}{100}$ = _____%

- Complete the sentence to find what fraction and what percentage of each bar model has been shaded.



_____ parts out of 10 = $\frac{\square}{10}$ = _____%



Complete the sentences to convert each fraction to a percentage.

Use the fraction wall to help you.

▶ $\frac{1}{2}$ ▶ $\frac{1}{4}$ ▶ $\frac{1}{5}$ ▶ $\frac{1}{10}$

$\frac{\square}{\square}$ = 100% split into _____ equal groups.

100 ÷ _____ = _____

So $\frac{\square}{\square}$ = _____%

- $\frac{1}{5}$ is equal to 20%.

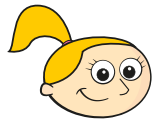
This means that $\frac{2}{5}$ is equal to 40%.

Complete the statements.

▶ $\frac{3}{5}$ = _____% ▶ $\frac{\square}{4}$ = 75% ▶ $\frac{7}{10}$ = _____% ▶ $\frac{\square}{5}$ = 80%

Percentages as fractions

Reasoning and problem solving



To convert a fraction to a percentage, you just need to put a per cent sign next to the numerator.

Is Eva correct?

Explain your answer.

No

This only works when the denominator is 100, because “per cent” means parts per hundred.

At a cinema, $\frac{4}{10}$ of the audience are adults.

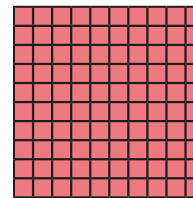


The rest of the audience is made up of boys and girls.

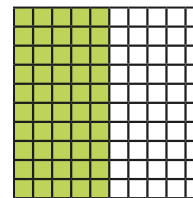
There are twice as many girls as boys.

What percentage of the audience are girls?

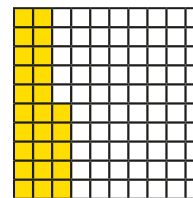
40%



$$100\% = 1$$



$$50\% = \frac{1}{2}$$



$$25\% = \frac{1}{4}$$

$\frac{1}{8}$ cannot convert to a percentage because 8 is not a factor of 100



Do you agree with Teddy?

Explain your answer.

No

Percentages as decimals

Notes and guidance

In the previous step, children began looking at the relationship between percentages and fractions. In this small step, they find decimal equivalents to percentages.

Use place value counters, bead strings and straws to recap that when 1 whole is split into 10 equal parts, each part is equal to 0.1 and when it is split into 100 equal parts, each part is equal to 0.01. Children relate this understanding to percentages, comparing 0.1 and 10%, and 0.01 and 1%. If $10\% = 0.1$ and $1\% = 0.01$, then $11\% = 0.1 + 0.01 = 0.11$

Children may begin to see a “trick” of writing “zero point” in front of the percentage to make a decimal, but this will cause confusion when converting single-digit percentages into decimals or, later, percentages greater than 100%. Exploring the equivalence of 0.01 and 1% using a variety of representations will help children avoid this misconception.

Things to look out for

- Children may see single-digit percentages as tenths rather than hundredths, for example $6\% = 0.6$
- Children may confuse percentages and decimals, for example $\frac{1}{2} = 0.50\%$

Key questions

- What is similar/different about percentages and decimals?
- How many tenths/hundredths/per cent are equal to 1 whole?
- What percentage is equal to one hundredth?
What is one hundredth as a decimal?
- What percentage is equal to one tenth?
What is one tenth as a decimal?

Possible sentence stems

- _____ = _____%
- There are _____ tenths/hundredths in 1 whole.
- _____% is equivalent to 1 whole.

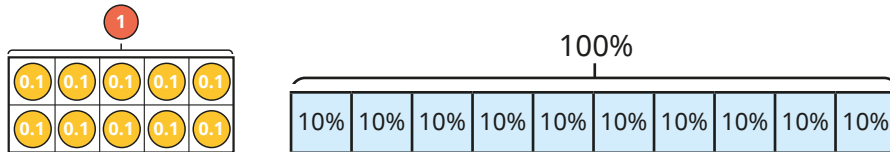
National Curriculum links

- Recognise the per cent symbol (%) and understand that per cent relates to “number of parts per 100”, and write percentages as a fraction with denominator 100, and as a decimal fraction
- Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

Percentages as decimals

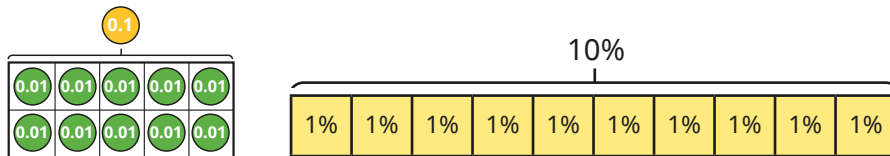
Key learning

- Use the models to complete the statements.



- ▶ $0.1 = \underline{\quad\quad\quad}\%$ ▶ $\underline{\quad\quad\quad} = 30\%$
- ▶ $0.8 = \underline{\quad\quad\quad}\%$ ▶ $\underline{\quad\quad\quad} = 100\%$

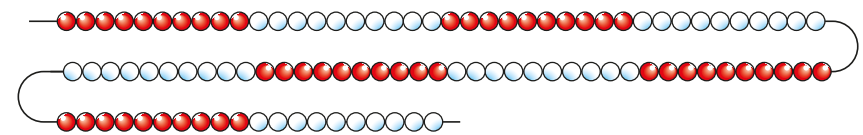
- Dora has used place value counters and a bar model to show that 0.01 is equivalent to 1%.



Use Dora's fact to complete the statements.

- ▶ $0.01 = \underline{\quad\quad\quad}\%$ ▶ $\underline{\quad\quad\quad} = 7\%$
- ▶ $0.05 = \underline{\quad\quad\quad}\%$ ▶ $\underline{\quad\quad\quad} = 9\%$

- Mo uses a 100-piece bead string to represent 100%.



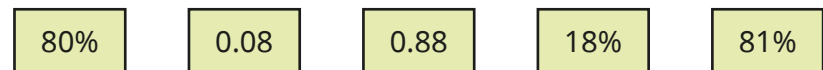
Complete the statements.

- ▶ 3 beads = $\underline{\quad\quad}$ = $\underline{\quad\quad}\%$
- ▶ 13 beads = $\underline{\quad\quad}$ = $\underline{\quad\quad}\%$
- ▶ 97 beads = $\underline{\quad\quad}$ = $\underline{\quad\quad}\%$
- ▶ $\underline{\quad\quad}$ beads = $\underline{\quad\quad}$ = 21%

- Write $<$, $>$ or $=$ to complete the statements.

$90\% \bigcirc 0.9$ $8.5 \bigcirc 85\%$
 $1\% \bigcirc 0.1$ $50\% \bigcirc 0.5$

- Write the decimals and percentages in ascending order.

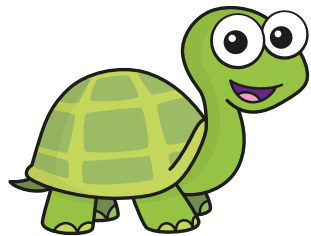


Percentages as decimals

Reasoning and problem solving

Tiny is comparing a percentage with a decimal.

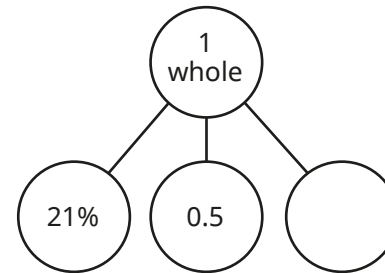
51% must be greater than 0.5 because 51% is more than half of 100% and 0.5 is exactly half of 1 whole.



Do you agree with Tiny?
Explain your answer.



Yes



What is the missing part?

Give your answer as a decimal and as a percentage.

0.29
29%

Using the digit cards only once for each solution, complete the comparison in as many different ways as you can.



$$0.\underline{\quad} < \underline{\quad}\% < \frac{3}{5}$$



multiple possible answers, e.g.
0.3 and 45%
0.46 and 53%

Compare answers with a partner.



Equivalent fractions, decimals and percentages

Notes and guidance

This small step builds on the previous two steps, with children now finding equivalent fractions, decimals and percentages. As this concept is covered again in Year 6, the focus at this stage should be kept quite narrow, mainly looking at the equivalents to halves, quarters, fifths and tenths. All of these equivalents can be found by splitting up a hundred square or bead string into the given equal parts and then making the link to hundredths.

Once children are confident finding the unit fraction equivalents, they explore finding the non-unit fraction equivalents, for example $\frac{3}{4}$, $\frac{2}{5}$ and $\frac{7}{10}$. Other representations, such as number lines and bar models, are useful for helping children to visualise the relationship between fractions, decimals and percentages. Children begin to explore less standard conversions such as 92%, which will be covered further in Year 6

Things to look out for

- If children do not have a secure understanding of the concept that the whole can be made up of 100 parts, some common errors can occur, particularly when converting fractions to percentages, for example writing $\frac{1}{5}$ as 5% or $\frac{7}{10}$ as 7%.

Key questions

- How can you find the fraction equivalent of a percentage?
- How can you find the decimal equivalent of a percentage?
- How many parts has the whole been split up into?
So what fraction is each part worth?
- If the whole is 100%, what is $\frac{1}{10}$?
- If $\frac{1}{10}$ is equal to 10%, what is $\frac{3}{10}$ equal to?

Possible sentence stems

- The whole has been split into _____ equal parts, so each part is worth $\frac{1}{\square}$
- If the whole is equal to 100%, then each part is worth _____%.

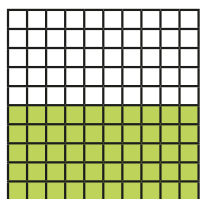
National Curriculum links

- Recognise the per cent symbol (%) and understand that per cent relates to “number of parts per 100”, and write percentages as a fraction with denominator 100, and as a decimal fraction
- Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

Equivalent fractions, decimals and percentages

Key learning

- $\frac{1}{2}$ of the hundred square is shaded.



$\frac{50}{100}$ is shaded.
0.5 is shaded.
50% is shaded.

Shade a hundred square and complete the sentences for each fraction.

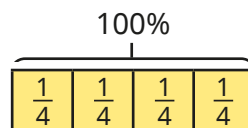
- ▶ $\frac{1}{5}$ $\frac{\square}{100}$ is shaded.
- ▶ $\frac{1}{10}$ _____ is shaded.
- _____ % is shaded.

Compare answers with a partner.

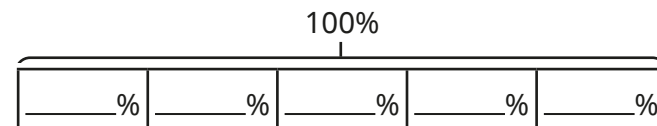
- What are the fraction and decimal equivalents of 92%?
What are the percentage and decimal equivalents of $\frac{28}{100}$?

- Use the bar model to help you complete the equivalence statements.

▶ $\frac{1}{4} = \text{_____} \% = \text{_____}$ ▶ $\frac{\square}{\square} = 75\% = \text{_____}$

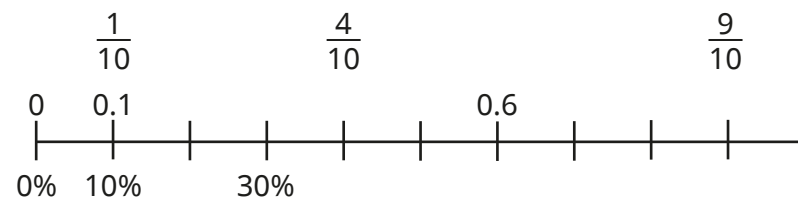


- Complete the bar model to help find the equivalents.



▶ $\frac{3}{5} = \text{_____} \% = \text{_____}$ ▶ $\frac{\square}{\square} = 40\% = \text{_____}$
 ▶ $\frac{\square}{\square} = \text{_____} \% = 0.8$ ▶ $\frac{\square}{\square} = \text{_____} \% = 1$

- Complete the number line to show the equivalents.



- Filip buys a bag of sweets.
He eats 70% of the sweets and gives $\frac{1}{10}$ to his sister.
What percentage of the sweets is left in the bag?
What fraction is left?

Equivalent fractions, decimals and percentages

Reasoning and problem solving

Are the statements true or false?



$$\frac{1}{10} = 10\%, \text{ so } \frac{1}{5} = 5\%$$

$$0.5 < 25\% \text{ because } 5 \text{ is less than } 25$$

$$\frac{1}{2} = 0.5 = \frac{2}{4} = 50\% = \frac{5}{10}$$

$$\frac{2}{5} = 0.4 = 4\%$$

Explain your reasons.



False
False
True
False

$\frac{1}{4}$ of the children in a class have brown hair.

$\frac{3}{5}$ have blonde hair.

15% have ginger hair.

How many children have black hair?

I cannot work out how many children have black hair because I do not know how many children are in the class altogether.



Do you agree with Tiny?

Explain your answer.

No

None of the children have black hair, because $\frac{1}{4} = 25\%$, $\frac{3}{5} = 60\%$ and $25\% + 60\% + 15\% = 100\%$

Spring Block 4

Perimeter and area

Small steps

Step 1

Perimeter of rectangles

Step 2

Perimeter of rectilinear shapes

Step 3

Perimeter of polygons

Step 4

Area of rectangles

Step 5

Area of compound shapes

Step 6

Estimate area



Perimeter of rectangles

Notes and guidance

In this small step, children build on learning from earlier years to find the perimeters of rectangles by measuring the sides and by calculation.

Children know that the perimeter is the distance around the outside of a two-dimensional shape. They recap measuring skills and recognise that they need to use a ruler accurately in order to get the correct answer. A common mistake is to measure from the end of the ruler rather than from the zero mark.

Children then explore different methods of finding the perimeter, for example adding all four sides separately, adding the length to the width and then doubling, or doubling the length and the width and then adding the results, before deciding which they find most efficient. Children use their understanding of perimeter to calculate missing lengths.

Things to look out for

- Children may line up the object they are measuring with the end of the ruler rather than the zero mark.
- When given the length and width of a rectangle, children may just add the two amounts.
- When measuring sides on a rectangle, children may get different dimensions for sides that should be equal.

Key questions

- What does “perimeter” mean?
- If a rectangle has a perimeter of 16 cm, could its length be 10 cm? Why or why not?
- Once you have measured the sides, how do you work out the perimeter?
- If you know the length and width of a rectangle, do you need to measure the other two sides?
- Which method do you think is more efficient?

Possible sentence stems

- The length is _____ and the width is _____, so the perimeter is _____
- _____ + _____ + _____ + _____ = $2 \times$ _____ + $2 \times$ _____
- The perimeter of the rectangle is _____

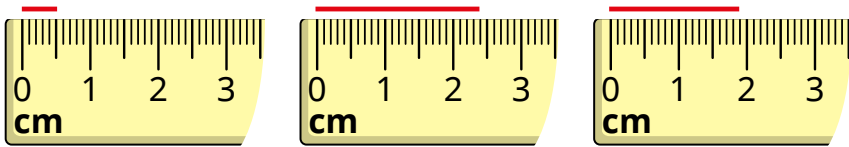
National Curriculum links

- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres

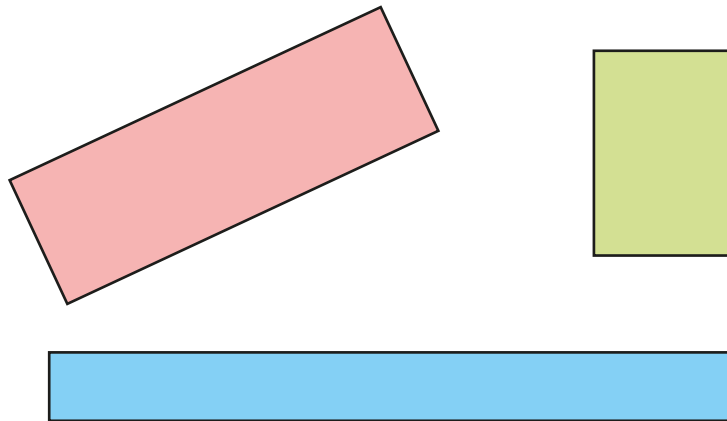
Perimeter of rectangles

Key learning

- What is the length of each line?



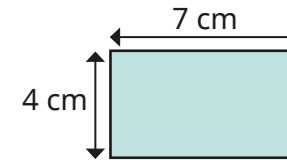
- Measure the sides of the rectangles to work out their perimeters.



_____ cm + _____ cm + _____ cm + _____ cm = _____ cm

- Draw a rectangle with a perimeter of 20 cm.
Compare your rectangle with a partner's.

- Rosie and Eva are finding the perimeter of this rectangle.



Rosie

$$7 \text{ cm} + 4 \text{ cm} + 7 \text{ cm} + 4 \text{ cm} = 22 \text{ cm}$$

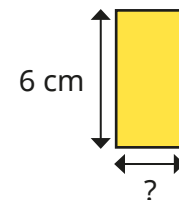
Eva

$$7 \text{ cm} + 4 \text{ cm} = 11 \text{ cm} \quad 11 \text{ cm} \times 2 = 22 \text{ cm}$$

What is the same about the methods? What is different?
Use both methods to find the perimeter of the rectangle.



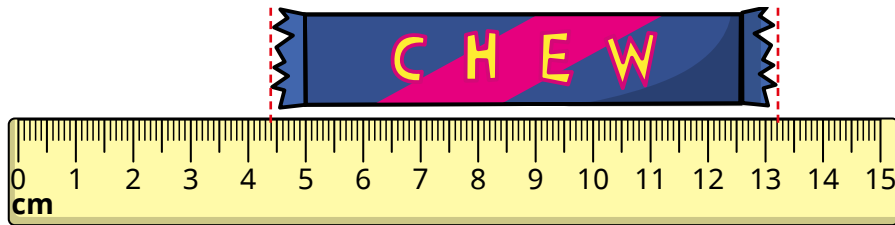
- The perimeter of a square is 16 cm.
What is the length of each side?
- The perimeter of this rectangle is 18 cm.
What is the width of the rectangle?



Perimeter of rectangles

Reasoning and problem solving

Teddy thinks this chew bar is 13.2 cm long.



Do you agree?

Explain your answer.



No

Is the statement always true, sometimes true or never true?

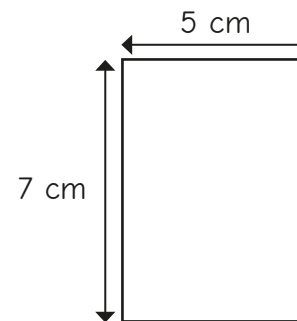
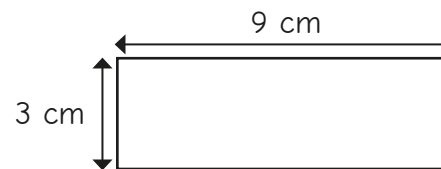
When the sides of a rectangle are all odd numbers, the perimeter is even.

Explain your answer.



always true

Esther thinks that she has drawn all the possible rectangles with a perimeter of 24 cm.



Do you agree with Esther?

Explain your answer.

No
multiple possible
answers, e.g.
A rectangle that is
11 cm by 1 cm has
a perimeter of
24 cm.

Perimeter of rectilinear shapes

Notes and guidance

In this small step, children build on their Year 4 learning to calculate the perimeters of rectilinear shapes.

A rectilinear shape is a shape that has only straight sides and right angles. This can look like two or more rectangles that have been joined together and is sometimes referred to as a compound shape. Children should be familiar with both terms. When calculating the perimeter of a rectilinear shape, encourage children to mark sides that they have already included in their total, to avoid counting sides more than once.

Children may notice the connection between the perimeter of some rectilinear shapes and the rectangle that can be drawn around the shape.

Things to look out for

- Children may miscount when adding the sides of rectilinear shapes.
- If children do not have a secure understanding of addition and subtraction, they may struggle when finding missing sides.
- Children may find it difficult to see that the two shorter sides are equal to the longer opposite side on the rectilinear shape.

Key questions

- What does “perimeter” mean?
- What are the properties of a square/rectangle?
- Why is this a rectilinear shape?
- How can you use the labelled sides to find the unknown side of the rectilinear shape? Do you need to add or subtract?
- What strategies can you use to work out the perimeter?
- How do you know that you have included all the sides?
- What is the perimeter of the shape?

Possible sentence stems

- _____ + _____ = _____, so the longer side = _____
- _____ - _____ = _____, so the other shorter side = _____
- The perimeter of the shape is _____

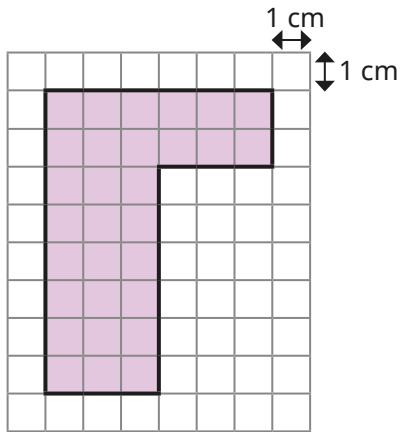
National Curriculum links

- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres

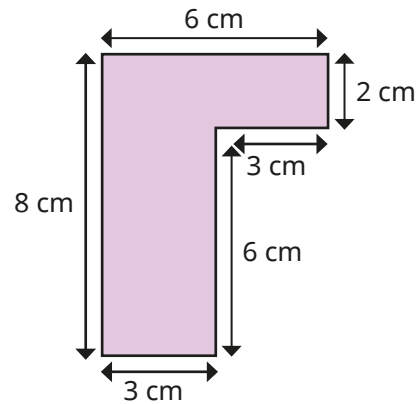
Perimeter of rectilinear shapes

Key learning

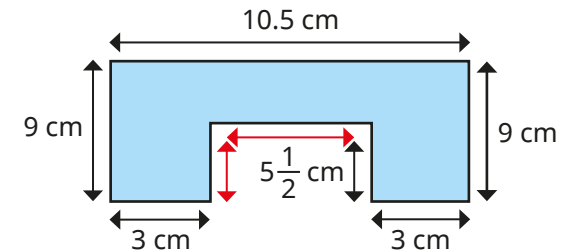
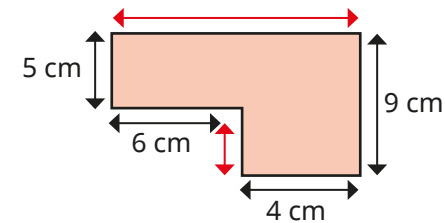
- Work out the perimeters of the shapes.



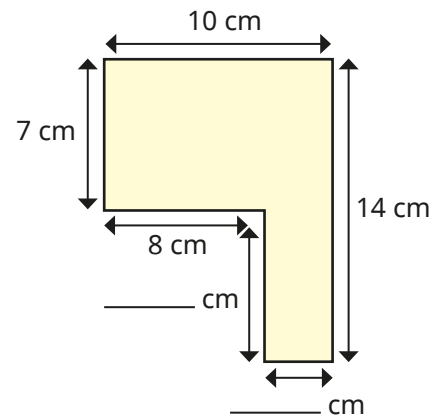
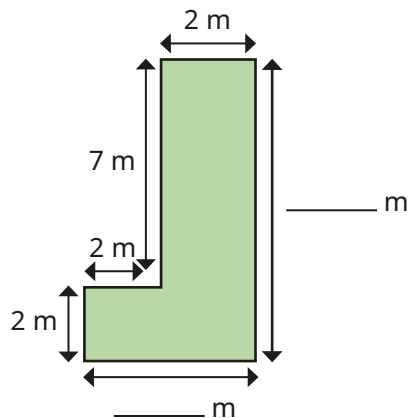
What do you notice?



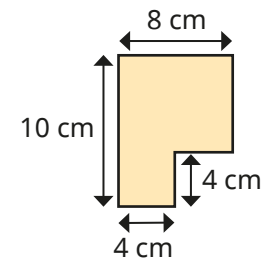
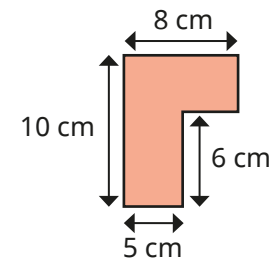
- Find the unknown lengths (shown in red) and then the perimeter of each shape.



- Work out the unknown lengths on each rectilinear shape.



- Work out the perimeters of the shapes.



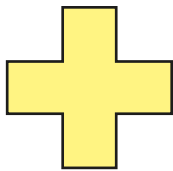
What do you notice?

Perimeter of rectilinear shapes

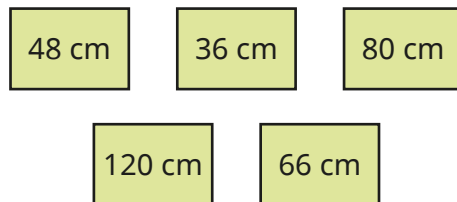
Reasoning and problem solving

Here is a rectilinear shape.

All the sides are the same length and are a whole number of centimetres.



Which of these lengths could be the perimeter of the shape?



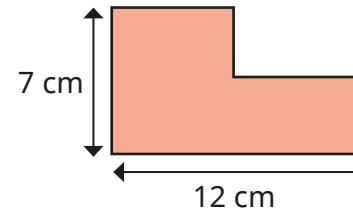
Explain your reasoning.

Can you think of any other possible perimeters?

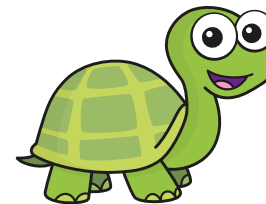
48 cm, 36 cm,
120 cm

any multiple of 12,
e.g. 24 cm, 72 cm

Tiny is finding the perimeter of this shape.



I have enough information to find the perimeter.



Do you agree with Tiny?

Explain your answer.

Yes

Perimeter of polygons

Notes and guidance

In this small step, children apply their knowledge of perimeter to find the perimeters of polygons and to solve word problems.

A polygon is a closed two-dimensional shape with straight sides. The difference between regular and irregular shapes could be a good discussion point during this step. A regular shape is a two-dimensional shape with equal sides and angles, so a square is a regular rectangle. When given the length of one side, children use their knowledge of regular shapes to find the perimeter by multiplying by the number of sides.

Children use the perimeter of a shape to find a missing side. Using pictorial representations, such as drawing the shape and adding the known values, will support children when problem solving.

Things to look out for

- Children may not be able to identify the relationship between the given length, width or perimeter in the problems.
- Children may confuse the terms “regular” and “straight” and think that all rectangles are regular.

Key questions

- What is a regular shape?
- What is the difference between a square and a rectangle?
- Are all rectangles regular?
- How many sides does the shape have? What calculation will give you its perimeter?
- Would drawing the shape help you to solve the problem?
- What operation are you going to use? Why?

Possible sentence stems

- A _____ shape has equal sides and angles.
- The regular shape has _____ sides and each side is _____
Therefore, the perimeter is _____ \times _____ = _____
- To find the perimeter of the shape, I need to...
- The perimeter of the shape is _____

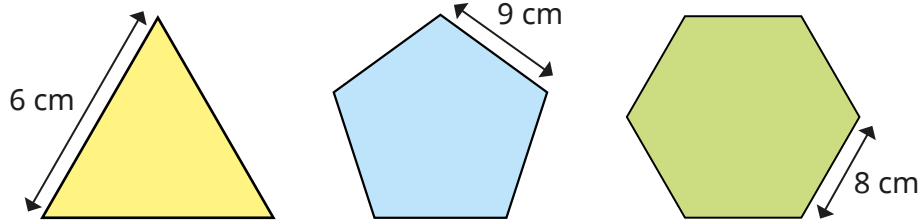
National Curriculum links

- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres

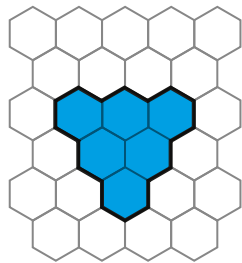
Perimeter of polygons

Key learning

- Work out the perimeter of each regular shape.

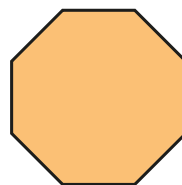


- Each regular hexagon on the grid has a side length of 2 cm.



What is the perimeter of the shaded shape?

- Mo measures three sides of this regular octagon. The total length of the three sides is 21 cm. What is the perimeter of the octagon?



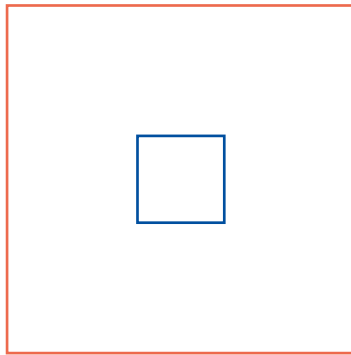
- The perimeter of a tennis court is approximately 70 m. Its width is 11 m. What is the length of the tennis court?
- A kitchen is 9 m long and 9 m wide. A living room has a perimeter of 38 m. Which room has the greater perimeter? What could the living room's length and width be?
- Tom wants to find the perimeter of a swimming pool. The length of the pool is three times the width. The width is 16 m. What is the length of the swimming pool? What is the perimeter of the swimming pool?
- The perimeter of a regular hexagon is 222 cm. Work out the length of one side of the hexagon.



Perimeter of polygons

Reasoning and problem solving

Here is a square inside another square.



16 cm

One side of the inner square is 4 cm long.

The perimeter of the outer square is four times the perimeter of the inner square.

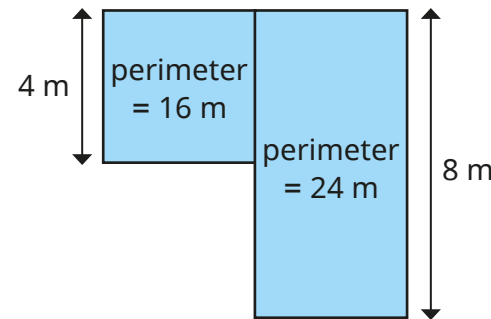
What is the length of one side of the **outer** square?

Show your workings.

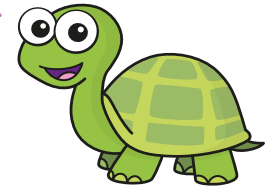
A school stage is made up of two parts.

The larger part has a perimeter of 24 m and a length of 8 m.

The smaller part has a perimeter of 16 m and a length of 4 m.



The perimeter of the stage is $24\text{ m} + 16\text{ m} = 40\text{ m}$.



Explain why Tiny is wrong.

Find the actual perimeter of the stage.

Tiny's total includes sides that are inside the shape.

32 m

Area of rectangles

Notes and guidance

In Year 4, children learnt that area was the space inside a two-dimensional shape. In this small step, they recap this key concept by making a visual comparison of two shapes without having to work out the area. They then go on to find the areas of shapes by counting squares, and are introduced to the square centimetre (cm^2) by counting squares on a centimetre squared grid. Highlight the difference between 1 cm and 1 cm^2 , to ensure children understand that cm is a measure of length and cm^2 is a measure of area.

Arrays can help children understand why they can multiply the length by the width to calculate the area of a rectangle, which they can then use to find the area of shapes not drawn on a centimetre squared grid. Children should be made aware that cm^2 is not the only unit used to measure area, and other units such as mm^2 , m^2 and km^2 are also examples of units of area.

Things to look out for

- When counting squares, children may count a square twice or miss a square out when counting.
- Children may rely on counting squares to find area, instead of multiplying the length by the width.
- Children may confuse the concepts of area and perimeter.

Key questions

- What is area?
- What is the difference between 1 cm and 1 cm^2 ?
- Which shape has the greater/greatest area?
Can you tell just by looking?
- How can you work out area in a more efficient way?
- Will multiplying the length by the width calculate the area of any shape? Why/why not?

Possible sentence stems

- There are _____ squares inside the shape, so the area of the shape is _____ squares.
- Area = _____ \times _____
- _____ \times _____ = _____, so the area of the shape is _____

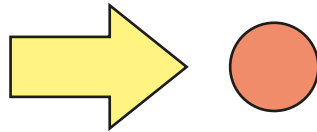
National Curriculum links

- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- Calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm^2) and square metres (m^2), and estimate the area of irregular shapes

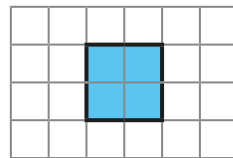
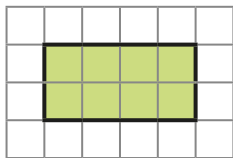
Area of rectangles

Key learning

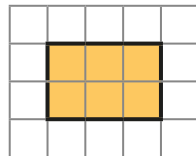
- Which shape has the greater area?
How do you know?



- On the grid, the area of each square is 1 cm^2
Find the area of each shape.



- Complete the sentences to find the area of the rectangle.



▶ There are _____ rows of _____ squares.

There are _____ squares altogether.

_____ \times _____ = _____

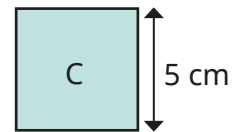
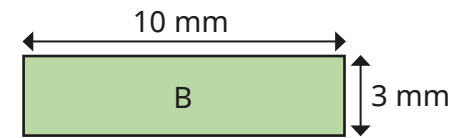
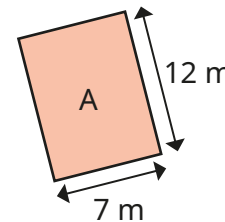
▶ There are _____ columns of _____ squares.

There are _____ squares altogether.

_____ \times _____ = _____

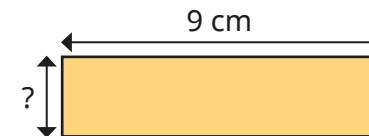
What do you notice?

- Shapes A and B are rectangles.
Shape C is a square.
Work out the area of each shape.



- Draw a rectangle with an area of 12 cm^2 and label the lengths.
How many different rectangles can you find?
They do not have to be drawn to scale.
Compare rectangles with a partner.

- The area of the rectangle is 18 cm^2



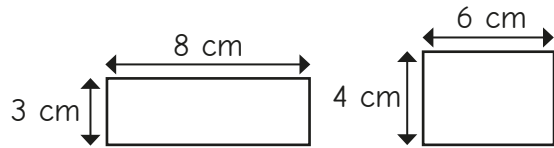
What is the width of the rectangle?

Area of rectangles

Reasoning and problem solving



Tiny thinks that these are the only rectangles that you can draw with an area of 24 cm^2



No

Do you agree with Tiny?

Explain your answer.



Is the statement always true, sometimes true or never true?



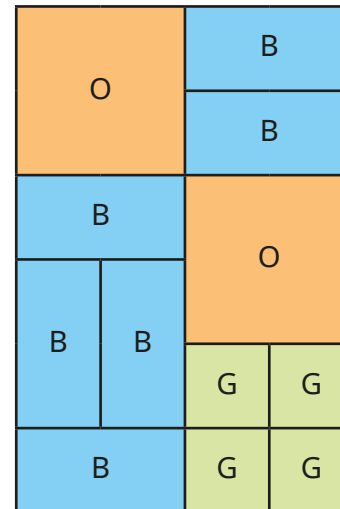
A rectangle's area is always greater than its perimeter.

sometimes true

Give examples to support your answer.



Each orange square (O) has an area of 24 cm^2



48 cm^2

72 cm^2

24 cm^2

144 cm^2

Calculate the total orange area.

Calculate the blue (B) area.

Calculate the green (G) area.

What is the total area of the whole shape?

Area of compound shapes

Notes and guidance

In this small step, children learn to calculate the areas of compound shapes, which are shapes made up of two or more other shapes. The focus is on rectilinear shapes.

To support their understanding, give children compound shapes for them to physically cut or split. They could find the area of each rectangle and deduce the total area of the shape. Some children will split their compound shape differently from others. This will highlight that a compound shape is made up from other shapes and that the area of the compound shape remains the same, whichever way the shape is split.

Children apply their learning from earlier steps to find missing lengths on the shape to support finding the area.

Things to look out for

- Children may rely on counting squares to find area, instead of multiplying the length by the width for the area of each rectangle.
- Children need to be secure in finding missing lengths of shapes by adding or subtracting known lengths.
- Children need to be careful when splitting up compound shapes to make sure they know which lengths correspond to which shape.

Key questions

- How do you work out the area of a rectangle?
- Are there any rectangles within the shape?
- How can you split the shape?
- Is there more than one way to split the shape?
- Do you get a different total area if you split the shape differently?

Possible sentence stems

- To find the area of the compound shape, I need to split it into _____ and then ...
- Area of rectangle A = _____
Area of rectangle B = _____
Total area = _____ + _____ = _____

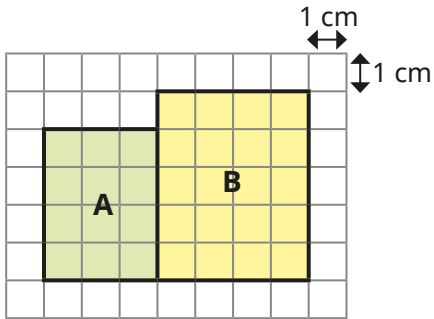
National Curriculum links

- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- Calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm²) and square metres (m²), and estimate the area of irregular shapes

Area of compound shapes

Key learning

- A compound shape is made up of two rectangles, A and B.

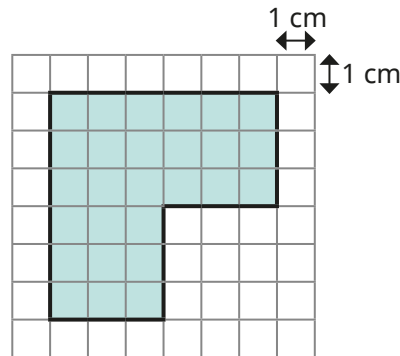


- ▶ What is the area of A?
- ▶ What is the area of B?
- ▶ What is the area of the compound shape?

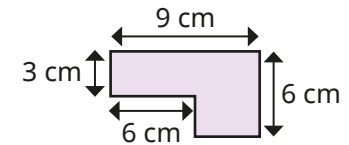
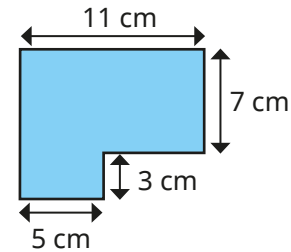
- Find the area of the compound shape.

How many ways can you split the compound shape in order to work out the area?

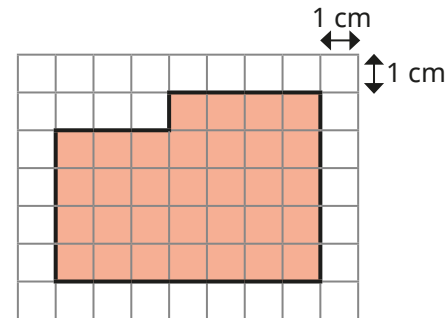
Compare methods with a partner.



- Find the areas of the compound shapes.



- Whitney has found the area of this compound shape.



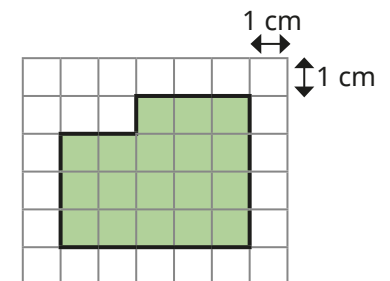
$$7 \times 5 = 35$$

$$35 - 3 = 32$$

The area is 32 cm^2

Explain why Whitney's method works.

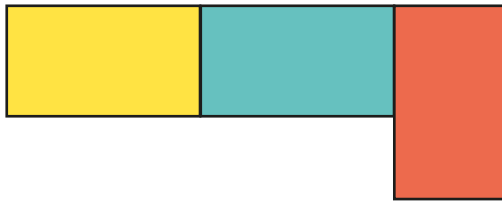
Use Whitney's method to find the area of the shape.



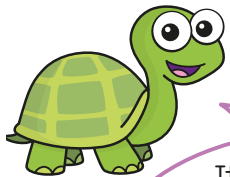
Area of compound shapes

Reasoning and problem solving

Tiny puts three 7 cm by 4 cm rectangles next to each other.



What is the area of the compound shape?



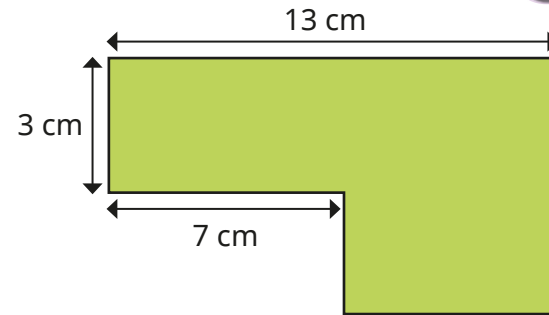
It does not matter which way round I put the rectangles. The shape will still have the same area.

Do you agree with Tiny?
Explain your reasoning.

84 cm²

Yes

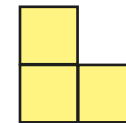
The area of the shape is 69 cm²



Work out the perimeter of the shape.

42 cm

The compound shape is made up of three squares.



The area of each square is 25 cm²
What is the perimeter of the compound shape?

40 cm

Estimate area

Notes and guidance

In this small step, children use their knowledge of counting squares to estimate the areas of non-rectilinear shapes.

Children should be aware that the estimate is not exact and other people may find a different estimate. One way to obtain an estimate is to find the total number of complete squares, then include a square if more than half of it is coloured, but not if less than half is coloured. Children use their knowledge of fractions to estimate how much of a square is covered.

For larger shapes, the areas of rectangles within them can be found by multiplying the length by the width, rather than counting all the squares individually.

To avoid repetition or miscounting, children can physically annotate when counting squares. An alternative method is to match up part-covered squares to create wholes, but this is more demanding and time consuming.

Things to look out for

- Children may struggle to identify which part-covered squares are more than half covered.
- Children may miscount or include the same square twice.

Key questions

- What does “approximate” mean?
- What does “estimate” mean?
- How many whole squares are covered?
- How many part squares are more than half covered?
- Are there any part-covered squares that you could combine to make a full square?
- Does it matter if your answer is not exactly the same as a partner’s? Why/why not?

Possible sentence stems

- _____ whole squares are covered.
- _____ squares are more than half covered.
- Estimate of the total area = _____ + _____ = _____ cm²

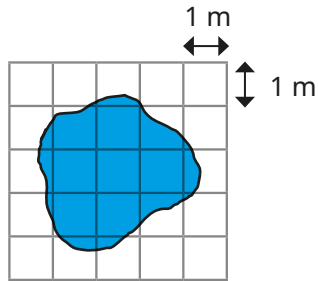
National Curriculum links

- Calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm²) and square metres (m²), and estimate the area of irregular shapes

Estimate area

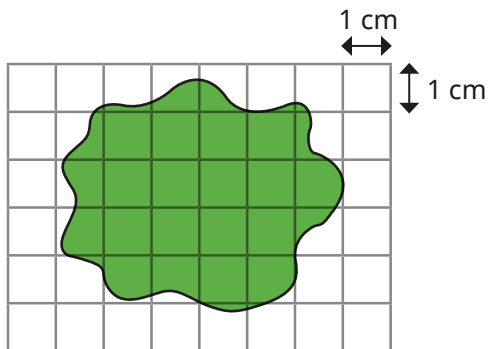
Key learning

- Jack estimates the size of the pond as 8 m^2



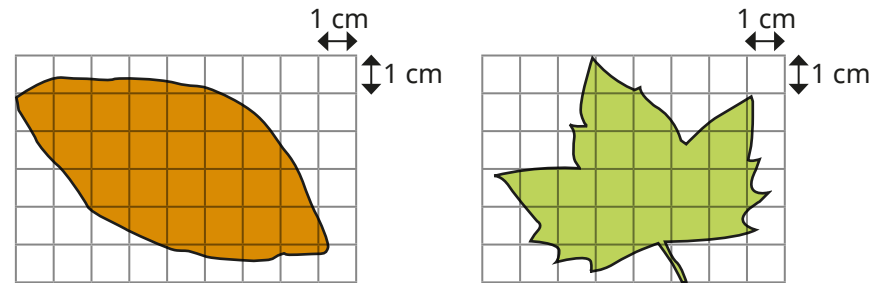
How do you think Jack made his estimate?

- Here is a shape on a centimetre squared grid.



- ▶ How many full squares are covered?
- ▶ How many squares are more than half covered?
- ▶ Estimate the area of the shape.

- Estimate the area of each leaf.



Which area was easier to estimate? Why?

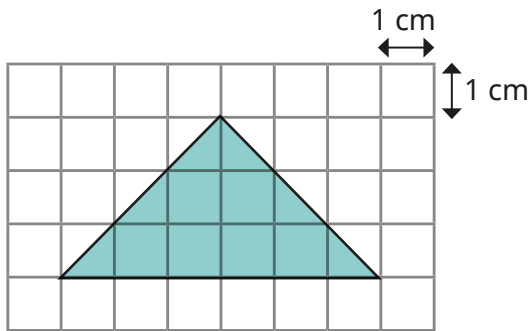
Compare answers with a partner.

- Draw a circle on centimetre squared paper.
Estimate the area of your circle.
Ask a partner to estimate the area of your circle.
Compare your estimates.
- Trace some other non-rectilinear shapes onto centimetre squared paper and estimate their areas.
Does where you put the shape on the grid make a difference to your estimate?
Compare answers with a partner.

Estimate area

Reasoning and problem solving

Amir is finding the area of the shape.



It is only possible to estimate the area of this shape.

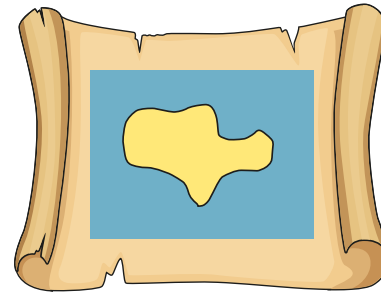


Do you agree with Amir?

Explain your answer



No



Use centimetre squared paper.

Draw a "Pirate Island" to be used as a treasure map.

Each square represents 4 m^2

The Pirate Island must have a total area of 248 m^2

The island must include these features:

- lake with a total area of 58 m^2
- forests with a total area of 86 m^2
- mountains with a total area of 92 m^2
- marshes with a total area of 12 m^2

Compare answers as a class.

Spring Block 5

Statistics

Small steps

Step 1

Draw line graphs

Step 2

Read and interpret line graphs

Step 3

Read and interpret tables

Step 4

Two-way tables

Step 5

Read and interpret timetables



Draw line graphs

Notes and guidance

In Year 4, children interpreted and drew line graphs for the first time, focusing on examples where the horizontal axis was a measure of time. In this small step, they revisit this learning and build upon it by looking at other types of graph, for example conversion graphs.

Encourage children to join points using a straight dashed line and discuss the fact that this is used because they cannot be certain of exact values between the given values at two points. However, this does not apply to conversion graphs.

Explore different sets of data that call for a range of intervals on the vertical axis. Children can decide what intervals to use by looking at the greatest and lowest values and using an appropriate scale.

Things to look out for

- Children may need support in choosing appropriate intervals for the vertical axis.
- Children may begin a scale from zero even if the lowest value is considerably greater than this.
- Children may not estimate accurately between two given values.

Key questions

- What information do you want to show with your line graph?
- What does the vertical/horizontal axis on the graph represent?
- What information will go on which axis? Why?
- Will you join the points with a solid line or a dashed line? Why?
- What scale would be most appropriate for the vertical axis?
- How can you use multiples to support your choice of intervals for the vertical axis?

Possible sentence stems

- The horizontal axis shows _____
The vertical axis shows _____
- The intervals on the vertical axis go up in _____

National Curriculum links

- Solve comparison, sum and difference problems using information presented in a line graph

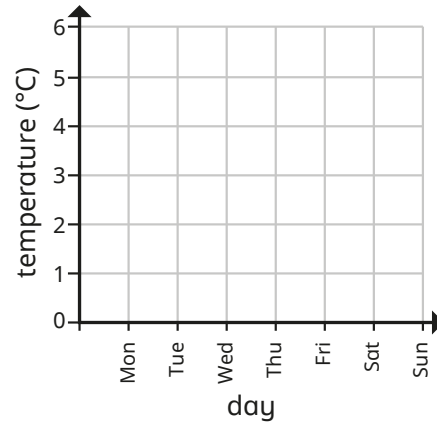
Draw line graphs

Key learning

- Scott records the temperature every day for a week.

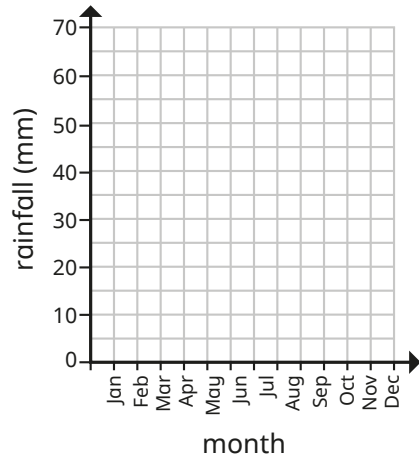
Use his results to draw the line graph.

| Day | Temperature (°C) |
|-----------|------------------|
| Monday | 2 |
| Tuesday | 3 |
| Wednesday | 3 |
| Thursday | 5 |
| Friday | 4 |
| Saturday | 2 |
| Sunday | 1 |



- The table shows the average rainfall in Leicester over a year.

Draw the graph using the information from the table.



| Month | Rainfall (mm) | Month | Rainfall (mm) |
|-------|---------------|-------|---------------|
| Jan | 55 | Jul | 69 |
| Feb | 45 | Aug | 64 |
| Mar | 49 | Sep | 58 |
| Apr | 57 | Oct | 63 |
| May | 60 | Nov | 61 |
| Jun | 66 | Dec | 60 |

- The table shows the average temperature for each month in Halifax.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Temperature (°C) | 4 | 4 | 5 | 8 | 10 | 15 | 17 | 16 | 13 | 11 | 6 | 5 |

Draw this information as a line graph.

- Dora measures her shadow in the playground every hour and records her results.

| Time | 9 am | 10 am | 11 am | noon | 1 pm | 2 pm | 3 pm |
|-----------------------|------|-------|-------|------|------|------|------|
| Length of shadow (cm) | 125 | 113 | 82 | 53 | 69 | 108 | 132 |

Draw the line graph for the data.

Start the vertical axis at 50

- Here is a table showing the conversion between pounds and Indian rupees.

| | | | | | | | | | | |
|--------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Pounds | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Rupees | 80 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 | 800 |

Present the information as a line graph.

What do you notice about the graph?

Draw line graphs

Reasoning and problem solving

Collect your own data and present it as a line graph.



You could collect data linked to a Science investigation.

Possible investigations could be:

- measuring shadows over time
- melting and dissolving substances
- plant growth

multiple possible answers

Here is a table of data.



| | | | | | |
|----------------|----|----|----|----|----|
| Time (minutes) | 15 | 30 | 45 | 60 | 75 |
| Distance (km) | 25 | 46 | 67 | 72 | 98 |

What intervals would be most appropriate for the vertical axis of the line graph?

Explain your answer.



multiple possible answers, e.g.
starting from zero, go up in 10s or 20s
starting from 20, go up in 10s

The chart shows the change in population of a village over 7 years.



| | | | | | | | |
|------------|------|------|------|------|-------|-------|-------|
| Year | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 |
| Population | 562 | 105 | 243 | 498 | 1,287 | 2,950 | 2,689 |

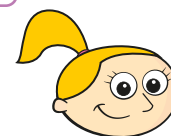
Mo, Eva and Rosie are turning the information into a line graph.



Mo

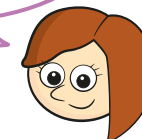
The intervals on the vertical axis should go up in 2s.

The intervals on the vertical axis should go up in 1,000s.



Eva

The intervals on the vertical axis should go up in 200s.



Rosie

Who do you agree with? Why?

Draw the line graph.



Eva

Read and interpret line graphs

Notes and guidance

In the previous step, children drew their own line graphs. In this small step, they interpret information that has been presented on a line graph and answer questions and solve problems using them.

Children read the graph at specific points to get information about one variable based on the other. They also find the difference between two points, the amount of time spent above/below certain points and make inferences based on information presented to them. Model questions such as the difference between two points by drawing straight lines between the graph points and the axis and then reading the scales accordingly.

Children should also explore estimating points between two intervals and should be able to explain why these are only estimates.

Things to look out for

- Children may not draw straight lines from the axis to the graph when reading off, so give inaccurate answers “by eye”.
- Children may choose an inappropriate estimate when the point is between two intervals.

Key questions

- What information is being presented on the line graph?
- What does each axis on the line graph show?
- How can you summarise what the graph shows?
- What lines can you draw to help read the graph?
- Why do you think the direction of the line changes at this point in the line graph?
- Is your answer exact or an estimate?

Possible sentence stems

- The horizontal axis shows _____ and the vertical axis shows _____
- At _____, the graph reads _____
At _____, the graph reads _____
The difference between the two points is _____

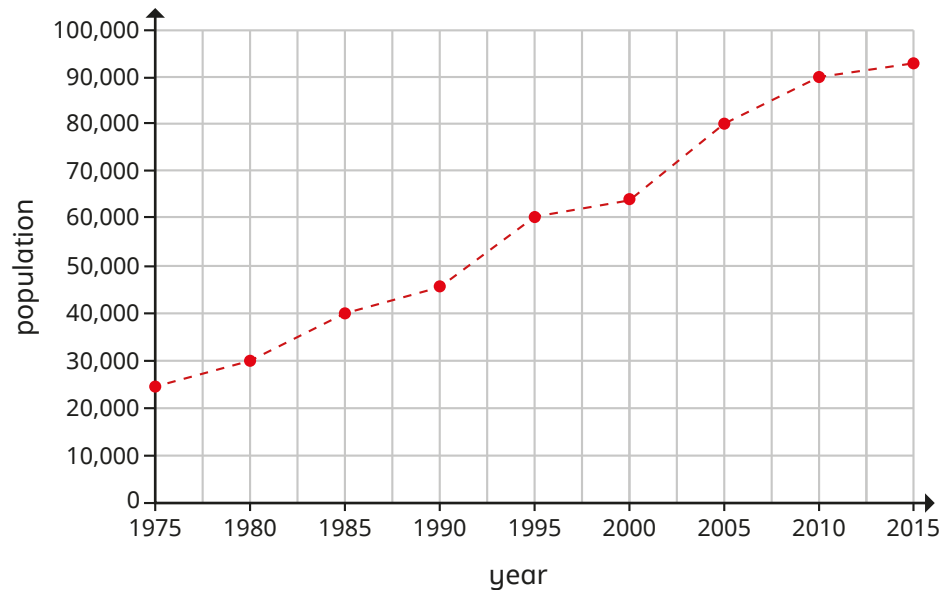
National Curriculum links

- Solve comparison, sum and difference problems using information presented in a line graph

Read and interpret line graphs

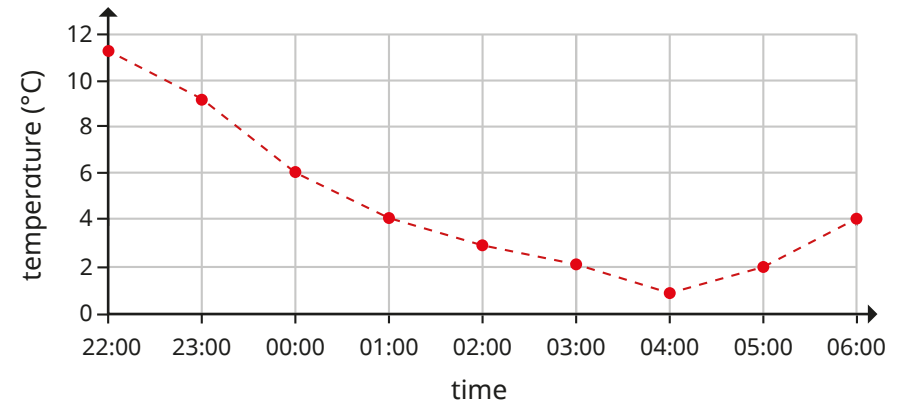
Key learning

- The line graph shows the population growth of a town.



- ▶ In what years was the population recorded?
How do you know?
- ▶ What was the population in 1985?
- ▶ What year did the population reach 80,000?
- ▶ Is it possible to know the exact population in 1997? Why?
- ▶ Estimate the year that the population reached 50,000
- ▶ Estimate the population in 2003

- The graph shows the night-time temperatures in a garden.

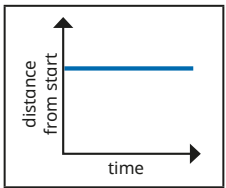


- ▶ How often was the temperature recorded?
How do you know?
- ▶ What was the temperature at midnight?
- ▶ Is it possible to tell the exact temperature at 02:30? Why?
- ▶ What was the highest recorded temperature?
At what time did this temperature happen?
- ▶ What was the lowest recorded temperature?
At what time did this temperature happen?
- ▶ What is the difference between the highest and the lowest temperature?
- ▶ What else can you find out?

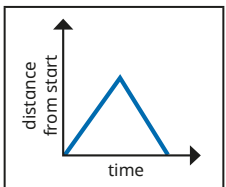
Read and interpret line graphs

Reasoning and problem solving

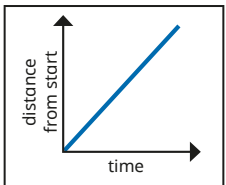
Match the graphs to the descriptions.



A car travels at a constant speed on the motorway.



A car is parked outside a house.

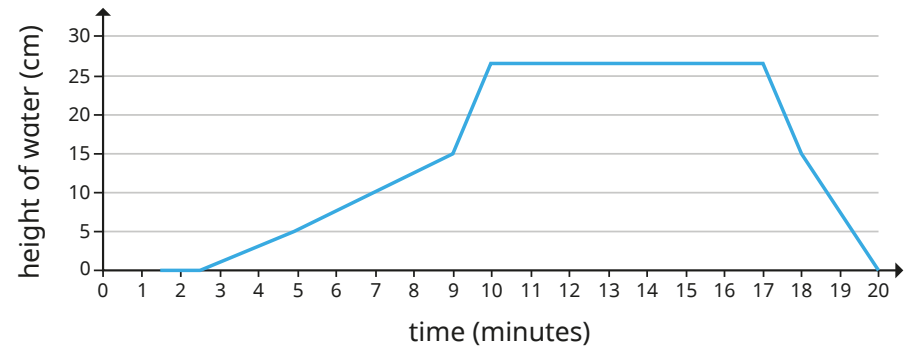


A car drives to the end of the road and back.

Explain your answers.

first graph,
second statement
second graph,
third statement
third graph,
first statement

The line graph shows the level of water in a bath.
Write a story to explain what is happening in the graph.



How long did it take to fill the bath?

How long did it take to empty?

The bath does not fill at a constant rate.

How does the graph show this?

Why might this be the case?

approximately 10 minutes

approximately 3 minutes

Read and interpret tables

Notes and guidance

In this small step, children read and interpret data presented in a table. They look at the data in a table and work out the information that they need to extract from the table to answer questions on the data. Look at a range of questions that can be asked about information in a table, beginning with simple retrieval questions and moving on to comparing amounts, inferring reasons behind information and grouping information. Encourage children to generate their own questions that can be answered using the table.

This step is a good opportunity for children to practise their addition and subtraction skills, as well as making comparisons.

This learning can be linked to Science and topic work.

Things to look out for

- Children may use the incorrect operation when answering questions about a table, especially for questions such as “How many more ... ?”
- Tables with more than two categories of information can be harder to interpret.

Key questions

- What information is given in this table?
- What are the column/row headings of the table?
- Why is it important to include the units of measure in the table?
- What is the total of _____?
- How can you find the difference between two pieces of information given in the table?
- How is a table similar to/different from a line graph?

Possible sentence stems

- The value in _____ is _____
The value in _____ is _____
The difference between the values is _____
- The _____ with the most/least _____ is _____

National Curriculum links

- Complete, read and interpret information in tables, including timetables

Read and interpret tables

Key learning

- Mo collects information from children about their favourite colour. He puts the information into a table.

| Colour | Red | Yellow | Green | Blue | Orange | Purple |
|--------------------|-----|--------|-------|------|--------|--------|
| Number of children | 3 | 7 | 5 | 17 | 6 | 7 |

- ▶ How many children prefer orange?
- ▶ What is the most popular colour?
- ▶ What is the least popular colour?
- ▶ How many children did Mo ask?
- ▶ How many more children like purple than like green?

What other questions could you ask about this table?

- Use the table to answer the questions.

| City | Leeds | Wakefield | Bradford | Liverpool | Coventry |
|------------|---------|-----------|----------|-----------|----------|
| Population | 720,000 | 316,000 | 467,000 | 440,000 | 305,000 |

- ▶ What is the difference between the highest and lowest populations?
- ▶ Which two cities have a combined population of 621,000?
- ▶ How much larger is the population of Liverpool than Coventry?

- Use the table to answer the questions.

| City | London | Sydney | New York | Reykjavik | Tokyo |
|--------------------------|--------|--------|----------|-----------|-------|
| January temperature (°C) | 7 | 27 | 2 | 0 | 10 |
| July temperature (°C) | 21 | 17 | 30 | 13 | 30 |

- ▶ In which city is the difference in temperature between January and July greatest?
- ▶ How much warmer is New York in July than Reykjavik in January?

- Here is a table with information about four planets.

| Planet | Time for revolution | Diameter (km) | Time for rotation |
|---------|---------------------|---------------|-------------------|
| Mercury | 88 days | 4,878 | 59 days |
| Venus | 225 days | 12,104 | 116 days |
| Earth | 365 days | 12,756 | 24 hours |
| Mars | 687 days | 6,794 | 25 hours |

- ▶ How many of the planets take more than one day to rotate?
- ▶ Which planet takes more than one year for one revolution?
- ▶ Write the diameter of Venus in words.
- ▶ What is the difference between the time for rotation of Mercury and the time for rotation of Earth?

Read and interpret tables

Reasoning and problem solving

The table shows some results from sports day.

| | 100 m sprint (seconds) | Shot-put (m) | 50 m sack race (seconds) | Javelin (m) |
|-------|------------------------|--------------|--------------------------|-------------|
| Amir | 15.5 | 6.5 | 18.9 | 11.2 |
| Dani | 16.2 | 7.5 | 20.1 | 13.3 |
| Teddy | 15.8 | 6.9 | 19.3 | 13.9 |
| Rosie | 15.6 | 7.2 | 18.7 | 14.1 |
| Ron | 17.9 | 6.3 | 18.7 | 13.3 |

Ron thinks that he won the 100 m sprint, because he has the greatest number.

Do you agree with Ron?

Explain your answer.

What other questions can you ask using the table?



No

The greatest number means the longest running time, so Ron is the slowest.

The table shows the six largest football stadiums in Europe.

| Stadium | City | Country | Capacity |
|---------------------------|-----------|---------|----------|
| Camp Nou | Barcelona | Spain | 99,365 |
| Wembley | London | UK | 90,000 |
| Signal Iduna Park | Dortmund | Germany | 81,359 |
| Estadio Santiago Bernabeu | Madrid | Spain | 81,044 |
| Luzhniki Stadium | Moscow | Russia | 81,006 |
| San Siro | Milan | Italy | 80,018 |

Are the statements true or false?

The fourth largest stadium is San Siro.

There is one stadium with a capacity greater than 90,000

Three of the largest stadiums are in Spain.

False True False

Two-way tables

Notes and guidance

In this small step, children explore two-way tables. Two-way tables show more than one piece of information about each variable, for example the number of adults and children in a school and how many do/do not wear glasses.

Start by looking at examples as a class, asking what information can be seen from the table. By generating their own questions, children will see the range of possible answers that a two-way table can show, identifying the meaning of each cell by looking at both the horizontal and vertical labels.

Children learn to find missing values in the table, such as the total number or one of the parts from given totals.

Things to look out for

- When finding the overall total, children may add the totals of the columns and the rows, and so find double the answer.
- Children may use the incorrect operation when finding missing numbers, for example adding instead of subtracting.
- Children may need support to identify the correct cell in a table that has the information they need.

Key questions

- What information is given by this table?
- What are the column/row headings of the table?
- How can you find the difference between two pieces of information given in the table?
- How can you work out missing information in the table?
- Do you need to add or subtract? How do you know?
- What conclusions can you draw from the table?

Possible sentence stems

- The columns show _____ and the rows show _____
- Where the _____ column meets the _____ row, this shows _____
- To find a missing total, I need to _____ the numbers in a _____ or _____
- To find a missing value, I need to _____ from _____

National Curriculum links

- Complete, read and interpret information in tables, including timetables

Two-way tables

Key learning

- The two-way table shows the staff at a police station.

| | No glasses | Glasses | Total |
|-----------------|------------|---------|-------|
| Constable | 55 | 24 | 79 |
| Sergeant | 8 | 5 | 13 |
| Inspector | 2 | 4 | 6 |
| Chief Inspector | 1 | 1 | 2 |
| Total | 66 | 34 | 100 |

- ▶ How many inspectors wear glasses?
- ▶ How many sergeants do not wear glasses?
- ▶ How many constables are there altogether?
- ▶ How many people work at the police station?

- The table shows information about type of pet and the pet's gender.

| | Male | Female | Total |
|-------|------|--------|-------|
| Dogs | | 44 | |
| Cats | 38 | | |
| Total | 125 | | 245 |

Fill in the missing numbers in the table.

- ▶ How many more male dogs are there than female dogs?
- ▶ How many more female cats are there than male cats?

- The table shows some information about how children in Key Stage 1 and Key Stage 2 travel to school each morning.

| | KS1 | KS2 | Total |
|-------|-----|-----|-------|
| Walk | | 95 | 118 |
| Car | 45 | | 70 |
| Bus | 9 | 27 | |
| Bike | | 56 | 56 |
| Total | | | |

- ▶ Complete the table.
- ▶ Which key stage has more children in it?
- ▶ What is the most popular method of getting to school for each key stage?

- The table shows the number of football matches won and lost by three different teams.

| | Liverpool | Manchester United | Chelsea | Total |
|-------|-----------|-------------------|---------|-------|
| Lost | 38 | 42 | 29 | |
| Won | 174 | 76 | 126 | |
| Total | | | | |

- ▶ Complete the table.
- ▶ Write some questions about the information for a partner to answer.

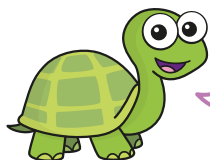
Two-way tables

Reasoning and problem solving

The table shows the types of sandwiches chosen by a group of children on a school trip.



| | White bread | Brown bread | Total |
|--------|-------------|-------------|-------|
| Ham | | 15 | 25 |
| Cheese | 13 | | 35 |
| Jam | | 8 | 17 |
| Tuna | 15 | | 23 |
| Total | | | |



$\frac{1}{5}$ of the children asked for a ham sandwich on white bread.

Do you agree with Tiny?
Explain your answer.



No

120 people were asked where they went on holiday during the summer months.



Use this information to create a two-way table.

- In June, 6 people went to France and 18 went to Spain.
- In July, 10 people went to France and 19 went to Italy.
- In August, 15 went to Spain.
- Altogether, 35 people went to France and 39 went to Italy.
- 35 people went away in June and 43 in August.

| | June | July | August | Total |
|--------|------|------|--------|-------|
| France | 6 | 10 | 19 | 35 |
| Spain | 18 | 13 | 15 | 46 |
| Italy | 11 | 19 | 9 | 39 |
| Total | 35 | 42 | 43 | 120 |

Read and interpret timetables

Notes and guidance

In this small step, children explore timetables, which are a special type of two-way table.

Start by showing children a timetable they are familiar with, such as their school day. Explain why it is important to have this information available and how anyone can read the timetable to understand information they may wish to know. Move on to other timetables that may be relevant to the children's lives, such as TV guides and timetables for local buses and swimming pools.

For this step, the questions will mainly focus on interpreting timetables.

Calculations using timetables will be covered in detail later in the year.

Things to look out for

- Children may assume that blank spaces need filling in, rather than understanding that buses or trains do not stop at that stop.
- Difficulties with times presented in digital form may hamper children interpreting timetables.

Key questions

- What information does this timetable tell you?
- How is a timetable the same as/different from a two-way table?
- What is the same and what is different about each row/column of the timetable?
- What does the _____ row/column tell you?
- At what time does the _____ from _____ get to _____?
- How many _____ are there?
- What does a blank space in a timetable mean?

Possible sentence stems

- The _____ train from _____ gets to _____ at _____
- The next available _____ is at _____
- The journey/lesson/programme starts at _____ and ends at _____

National Curriculum links

- Complete, read and interpret information in tables, including timetables

Read and interpret timetables

Key learning

- This is Alex’s school timetable.

| | | 1 09:15– 09:55 | 2 09:55– 10:45 | | 3 11:05– 11:55 | 4 11:55– 12:45 | | 5 13:45– 14:35 | 6 14:35– 15:25 |
|------|------------------------------|----------------------|----------------------|---------------------|----------------------|----------------------|-------------------------|----------------------|----------------------|
| Mon | Daily Assembly (09:00–09:15) | Literacy | English | Break (10:45–11:05) | Maths | ICT | Lunchtime (12:45–13:45) | PSHCE | Geog |
| Tue | | English | Art | | French | Science | | DT | |
| Wed | | Literacy | DT | | Art | Drama | | ICT | Science |
| Thur | | PE | Maths | | RE | English | | History | PSHCE |
| Fri | | Literacy | Maths | | Art | Science | | PE | |

- ▶ How many Literacy lessons does Alex have in a week?
- ▶ Which afternoons does she only have one subject?
- ▶ How many more Maths lessons does Alex have in a week than ICT lessons?
- ▶ At what time does Alex’s Science lesson on Friday start?

What other questions can you think of for Alex’s timetable?

- Here is part of a train timetable.

| | | | | |
|-----------------------|-------|-------|-------|-------|
| London Euston | 06:35 | 15:10 | 16:10 | 18:40 |
| Watford Junction | 06:50 | 15:25 | 16:25 | 18:55 |
| Milton Keynes Central | 07:10 | | 16:50 | |
| Northampton | 07:15 | 15:55 | 16:55 | 19:25 |
| Rugby | 07:24 | 16:04 | 17:04 | 19:34 |
| Coventry | 07:44 | 16:14 | 17:13 | 19:43 |
| Birmingham New Street | 08:09 | 16:41 | 16:41 | 20:11 |

- ▶ What time does the 15:10 train from London Euston get to Coventry?
- ▶ Annie gets on the train at Northampton. How many stops are there before she gets to Birmingham New Street?
- ▶ Ron gets a train from Watford Junction to Rugby. He arrives in Rugby at 16:04. What time did he get on the train?
- ▶ Why are some parts of the table blank?

Read and interpret timetables

Reasoning and problem solving

Here is part of a TV guide.



| 5 pm | | 6 pm | | | 7 pm | |
|-----------------|----------------|---------|-----------------|---------------|---------------|---------------|
| NatureWatch | News | Weather | Deep Blue | Pampered Pets | In the Wild | Safari |
| NatureWatch + 1 | Puppy Playtime | | News | Weather | Deep Blue | Pampered Pets |
| QuizTime | Talk the Talk | Quizdom | What's the Q? | aMAZEment | Buzzed Out | |
| CookeryPro | Cheese Please | | Cook with Lydia | Pizza Pro | 5 Minute Menu | Budget Baker |

Huan wants to watch *Cheese Please*, *Pampered Pets*, *aMAZEment* and *Budget Baker*.

Will Huan be able to watch all the programmes he has chosen?

Yes

Here is a bus timetable.



| | | | | |
|-------------------|-------|-------|-------|-------|
| Bus terminal | 09:32 | 10:02 | 10:22 | 10:32 |
| Shopping centre | 09:41 | 10:11 | 10:31 | 10:41 |
| Football stadium | 09:59 | 10:29 | 10:49 | 10:59 |
| University campus | 10:13 | 10:43 | 11:03 | 11:13 |
| Library | 10:16 | 10:46 | 11:06 | 11:16 |
| Cinema | 10:21 | 10:51 | 11:11 | 11:21 |
| Museum | 10:28 | 10:58 | 11:18 | 11:28 |

Sam lives a 15-minute walk from the bus terminal.

She wants to visit Whitney, who lives a 10-minute walk from the cinema.

She says she will meet Whitney at Whitney's house at 11:15

What time does Sam need to leave her house?

09:47